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The Performance of Multithreshold Decoder over Fading Channels

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***Abstract*—Multithreshold decoders (MTD) for self-orthogonal codes (SOC) are discussed. It's known MTD can provide near optimum decoding with linear complexity only. Bit-error rate performance of MTD in multipath Rayleigh and Rician fading channels is analyzed. Several questions of application of MTD with orthogonal frequency division multiplexing and multiple-input and multiple output technology for improving reliability of communication with unmanned vehicles are discussed. The performance of multithreshold decoders in such conditions is presented. Comparison with decoders for DVB-S2 low-density parity-check codes is presented. It's shown MTD provides comparable with LDPC decoders performance at much lower complexity.**

Keywords—fading channels, wireless communication system, error-correcting coding, self-orthogonal codes, mutithreshold decoders, unmanned vehicles

I. INTRODUCTION

Error correction codes are used for improve reliability of data transmitted over wired or wireless communication channels. Convolutional codes with Viterbi decoders [1], turbo codes [2], low-density parity-check codes [3] and other codes are used in modern communication systems now. However, these codes are still very complex for decoding or inefficient. This paper describes a least complex high-speed iterative decoder named multithreshold decoder (MTD). Each iterations of MTD is formed by the traditional threshold decoder (TD) [4] with a special difference register.

It's known the self-orthogonal codes (SOCs) are used with MTD give limited error propagation with TD and prevent catastrophic error flow [5]. However, the error performance of TD is not attractive. Known iterative decoders for SOC [6, 7] are more complex than TD. We had proposed improved version of iterative TD [8, 9] called MTD. Many publications on MTD [8..12] shows this decoder can provide near optimum decoding for long SOCs at high noise level in Gaussian channel with linear complexity. Moreover it's suggested several methods for MTD's performance improvement in such channel [9, 10, 13]. Very interesting results were obtained in field of multithreshold decoding of non-binary self-orthogonal codes [14]. Symbolical MTD and concatenated schemes based on it provide much better symbol error rate performance than known decoders for Reed-Solomon codes. This allows application of MTD for error correction in such high-speed communication channels as optical channels [15, 16].

It should be noted a distribution of errors in real radio channels due to multipath propagation, Doppler effect and other reasons is much more complex. It's known in such channels orthogonal frequency division multiplexing (OFDM), multiple-input multiple-output (MIMO) technologies and others are used with error-correction codes for increase reliability of communication. The research of MTD application with described technologies for several typical models of multipath fading channels is presented in the paper.

II. THE MULTITHRESHOLD DECODING

This section provides multithreshold decoding algorithm. Consider a binary linear systematic block or convolutional SOC with the rate $R = k/n$. The parity-check matrix for the code is set in systematic form: **H**=[**C** : **I**], where **I** is identity matrix and matrix **C** consists of 0 and 1.

Let's a codeword **A** generated with the usual SOC encoder [4] is transmitted over binary symmetric channel (BSC). The decoder receives vector Q=A⊕E. Here ⊕ is modulo 2 addition and E is vector of channel errors need to be corrected.

The MTD performs the following steps during vector Q decoding.

1. Syndrome S = **H**·Q is calculated. Information part of Q is placed into information register U. Difference register D is initially filled with zeros.

2. An information bit u_j for decoding is chosen. For this symbol checksum L_j is calculated:

$$L_j = \sum_{s_{j_k} \in \{S_j\}} s_{j_k} + d_j , \quad (1)$$

where $\{S_j\}$ is a set of checks corresponding to bit u_j and d_j is bit of the register D corresponding to the decoded bit u_j.

3. If checksum value $L_j > T$ (here $T=d/2$ is threshold value; d is the code distance), the decoding is done by flipping the information bit. At the same time related difference bit and syndrome bits are inverted. The register D will always show the difference between received and decoder information bits.

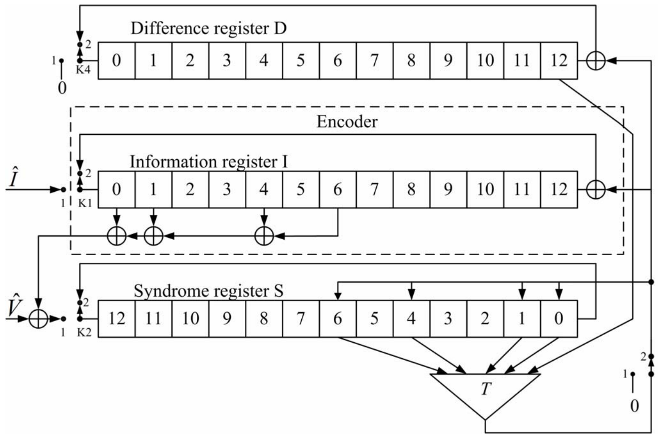

Fig. 1. An example of multithreshold decoder scheme for a block SOC

It should be noted after flipping each information bit the Hamming distance between the received signal and decoded codeword became shorter [8, 10].

4. If stopping criterion is not fulfilled, go to the step 2 with next information bit. Completing of some maximum number of decoding iteration may be one of possible stopping criteria.

It should be noted the MTD decisions after flipping of each information bit are strictly closer to the maximum likelihood decision even after many decoding iterations. Certainly, high MTD efficiency at high noise is available for special SOCs with low level of error propagation. This important issue is considered in [8, 10].

A scheme example of MTD for a block SOC of rate 1/2 with generator polynomial $g(x)=1+x+x^4+x^6$ is shown in fig. 1. The decoder contains several shift registers, modulo 2 adders and threshold element. These are the simplest elements for implementation. The MTD for a convolutional SOC has a similar simple structure.

The MTD algorithm was described for the binary symmetric channel. It's known using soft decisions provided by demodulator allows to increase coding gain about 2 dB in Gaussian channel. The MTD can be modified to use the soft decisions. The soft MTD calculates the checksum value from a set of the weights of parity bits related to the information bit under decoding and the weight of information bit itself. In this case the likelihood function L_j is calculates by

$$L_j = \sum_{s_{jk} \in \{S_j\}} w_{j_k}(2s_{j_k} - 1) + w_{d_j}(2d_j - 1) . \qquad (2)$$

Here w_{d_j} is weight of the information bit u_j and w_{j_k} is weight of parity bit. For the binary symmetric channel this expression with weights w_k=1 is equivalent to (1). For a Gaussian channel weight coefficients for calculation of L_j may be chosen as relatively small real or integer numbers. For example, w_{d_j} may be defined with reliability of received information bit r_j and w_{j_k} may be defined with reliability of received parity bit or with minimal reliability of bits are used for checksum value calculation or in other way. If checksum value $L_j > 0$ the decoding is done by flipping information bit and related difference bit and syndrome bits. After flipping each information bit, the Euclidean distance between the received signal and the decoded codeword becomes shorter [8, 10].

The discussed results shows the increasing number of attempts to correct information bits with the MTD is useful, as with every flipping of information bit there is transfer to more likelihood decision. Nevertheless, it does not mean that MTD has to achieve the maximum likelihood (ML) decision. For many codes there are many patterns of channel errors are corrected by an ML decoder but are not corrected by an MTD. It occurs due to the fact that the performance of an MTD is limited with the error propagation (EP) effect [8, 10]. The second and the rest iterations of MTD usually have to decode data with error groups from the previous decoding iterations.

A method of error propagation estimation for SOC was given in [8, 10]. The method is based on calculation of probabilities for single and packets errors at TD output with using multidimensional probability generating functions. Suggested method is helpful both for selection of SOC with low error propagation and for determination of optimum parameters of MTD ensuring the best performance. The analyze of error propagation in SOC shows a SOC with several information and several parity branches has a lower error grouping during iterative decoding and provides better bit error performance at high noise level.

III. DECODING COMPLEXITY

The main MTD advantage is very low complexity. The decoding complexity is defined by total number of additive equivalent operations necessary to decode an information bit.

At decoding of each information bit the MTD sums weighted checks. If the sum value is greater than a threshold, the information bit, related difference bit and checks are flipped. The number of decoding iterations I in this case usually does not exceed 50. So the MTD complexity is estimated for code of distance $d < 25$ by

$$N_1 \sim (d+2)(I+4) . \qquad (3)$$

Besides it is possible to decrease the count of operations as the information bits are very seldom changed during decoding. If it is possible to decrease the performance of MTD about 0,1 dB, the count of decoding operations may be estimated by

$$N_2 \sim c_1 d + c_2 I , \qquad (4)$$

where constants c_1 and c_2 are small integers [8, 10].

Known hardware MTD implementations, for example, on serial FPGA Xilinx or Altera, shows good bit error performance with simultaneously very high throughput up to 1,6 Gbps [17]. Such opportunity appears after the realization of patented engineering solutions for hardware MTD. MTD with these solutions is a single-cycle decision circuit and is able to make up to 40 decisions on the decoding bits at each decoding step. As a result data speed of hardware MTD can considerably exceed even 10 Gbps. Similar results on high-speed MTD were presented in [15, 16] for an optical communication systems.

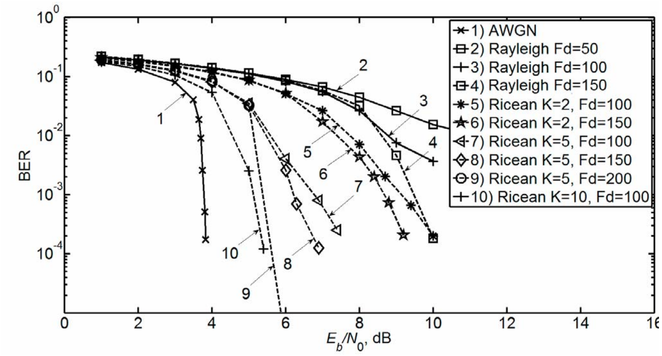

Fig. 2. Bit error performance of MTD over fading channels

IV. THE PERFORMANCE OF MTD OVER FADING CHANNEL

Lets further discuss the performance of MTD over different fading channels such as Rayleigh and Rician channels. In these cases the data modulation is considered as QPSK and demodulation is considered as hard-decision and no interleaving is used.

Fig. 2 shows simulation results of MTD for the SOC with code rate R=1/2, code distance d=17, code length n=32768 bits. The code is characterized with low error propagation and has 8 information and 8 parity branches. For improving waterfall region the parallel concatenation is used [10]. The maximum number of iterations is set to 30 for hard decoding MTDs. Curve *1* represents BER performance of MTD over the AWGN channel. Other curves for fading channels can be compared with curve *1*.

The performance of MTD for Rayleigh fading channel with Doppler shift 50, 100 and 150 Hz is shown in fig. 2 by curves *2*, *3* and *4* respectively. In this case scattered signals only arrive to receiver and there is no line of sight signal. It should be noted the performance of the MTD over Rayleigh fading channel is much worse than over AWGN channel. Also the performance for the MTD is better for channel with faster fading. It can be explained by fact that long error groups occurred in the channel with slow fading can't be corrected by the MTD as well as by other known error correction methods without using of long interleaver. The rest curves are correspond to the Rician channel when there is a line of sight signal between a transmitter and a receiver.

Curves *5* and *6* show the performance the MTD over Rician channel with the rician K-factor of 2 and Doppler shift 100 and 150 Hz. The MTD performance over channel with K-factor of 5 and Doppler shift 100, 150 and 200 Hz is reflected by curves *7*, *8* and *9* respectively. And finally, curve *10* represents the MTD performance for K=10 and F_d=100 Hz. The comparison of these curves shows the increasing of K-factor results to improving of the MTD performance. But even for K=10 the loss about 2 dB is observed in comparison with using of AWGN channel. Furthermore, the MTD performance is better for faster fading also.

It should be noted discussed case of MTD using allows to increase data transfer reliability over frequency non-selective channels only as Rayleigh and Rician channels. If there is frequency-selective fading (i.e. intersymbol interference is occurred), additional technologies have to be used. One of them is orthogonal frequency division multiplexing (OFDM). The OFDM uses multi-carrier modulation and has much lower symbol rate compared single carrier modulation. As a result the OFDM is robust against intersymbol interference and fading caused by multipath propagation.

V. A USING OF MTD FOR COMMUNICATION WITH UNMANNED VEHICLES

Wide using of unmanned vehicles, mobile robots for collection, processing and transfer of huge volume of information requests increased demands to communication system: very high speed of data transmitting, high dynamics of unmanned vehicle, multipath propagation and fading, real-time data transmission. In this condition complex using of high performance techniques for increasing data transmission reliability only enables to solve all assigned tasks.

For a multipath fading channel the most effective method to performance improvement is the spatial diversity (multiple-input multiple output technology – MIMO) allowing to greatly increase communication channel capacity. This technology is often used with OFDM and error correction coding to provide requester bit error performance and coding gain. In this condition coding and decoding have to be very simple for implementation for ensuring the demanded high communication rates up to hundreds megabits per second and even more. Fully the discussed multithreshold decoders for self-orthogonal codes meet these requirements.

The research of the MTD with MIMO and OFDM performance carried out with support of the Russian Science Foundation (grant №14-19-01263). In this research the data modulation is QPSK, channel model is Urban micro of Spatial Channel Model, OFDM with 1024 subcarriers with parameters from IEEE802.16e standard is used, and the length of guard interval is 1/16 of OFDM symbol length. For error correction the SOC with code rate R=1/2, code distance d=17, code length n=32768 bits and DVB-S2 LDPC code with code rate 0.44, code length 16200 bits were used.

Fig. 3 shows the bit error performance of the discussed decoders. Curves *1* and *2* show bit error performance of MTD for the SOC and min-sum decoder for the LDPC code respectively. These results were obtained for hard-decision demodulator. It should be noted the decoders for the codes provide similar bit error performance. A using of two transmit and two receive antennas (2x2 MIMO) decreases bit error performance for the SOC (curve *3*) and LDPC code (curve *4*) slightly, but bit rate in this case is increased in two times without bandwidth expansion. The same bit rate increasing turns out with the SOC and QAM16 modulation using (curve *5*), but bit error performance of this combination is about 2 dB worse than 2x2 MIMO using. Note additionally the using MIMO with the SOC gives some more gain than using MIMO with the LDPC code. It bit rate is not increasing (i.e. one transmitting antenna with QPSK is used), a using several receive antennas allows increase data transmission reliability greatly. Example of bit error performance for using two and three antennas with the SOC is shown in fig. 3 by curves *6* and

7 respectively. The gains in such cases are about 4 and 7 dB at BER 10^{-5} in comparison with one receive antenna. Similar improvement is available with using of the LDPC code with one transmitting and two receiving antennas (curve *8*). A using of QAM16 instead of QPSK with 1x2 MIMO and the SOC allows to get bit error performance are shown by curve *9*. This combination loses previously considered version with QPSK about 4 dB in coding gain, but bandwidth narrows two times. The fig. 3 shows additionally the attempts of further bit rate increasing with using more transmit antennas leads to

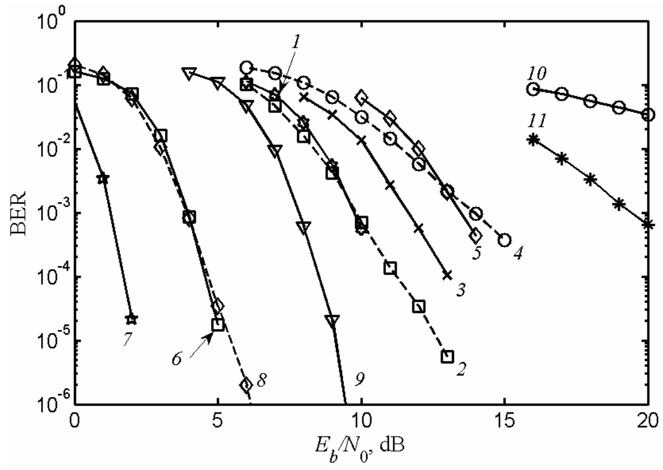

Fig. 3. Bit error performance of MTD over fading channels with MIMO

degrade bit error performance significantly (curve *10* for 4x4 MIMO with the SOC and QPSK and curve *11* for 2x2 MIMO with the SOC and QAM16). So an increasing bit rate with using more transmit antennas only is unsuitable, or it need to use some additional technologies as precoding allowed to locate data and parity symbols by antennas with using state channel information.

It should be noted these are only initial results on research the MTD performance over different fading channels. In future several tasks on using the MTD with OFDM, MIMO need to be solved. For example it is nee to determine of optimal information and parity symbols mapping on subcarriers of OFDM and on transmit antennas. It is interesting to evaluate influence of inaccuracy in channel state estimation and to research bit error performance of the MTD using with MIMO and space-time coding . The solution of these tasks provides to increase efficiency of communication systems significantly.

VI. Conclusion

The presented results show the MTD enables to provide high bit error performance over channels with different kinds of fading. In these conditions many other error correction methods are unable to work. It should be noted the coding gain for fading channels is many times more in comparison with Gaussian channel as for fading channels it is impossible to ensure data reliability requested even at cost of unlimited increasing of signal power. Additional advantage of MTD is very low implementation complexity. It enables to use MTD in high-speed communication systems.

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