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Increase Efficiency of Multilevel Multithreshold Decoder for Self-Orthogonal Codes

Cao V.T., Grinchenko N.N., Ovechkin G.V.,
Faculty of Computer Engineering
Ryazan State Radio engineering University
Ryazan, Russian Federation
g_ovechkin@mail.ru

***Abstract*—This paper considers multilevel multithreshold decoder for a self-orthogonal codes. Simulation results for an AWGN channel are reported. New modifications of this decoder provided coding gain improving by a few tenths of dB due to better using of the bits soft information are offered. Recommendations to choose the best algorithms of constituting blocks of the multilevel multithreshold decoder and its parameters are given.**

Keywords*—error-correcting coding, self-orthogonal code, multithreshold decoder, multilevel multithreshold decoder.

I. Introduction

The comparison of error-correcting codes decoding algorithms [1] shows that the most efficient algorithms like Viterbi algorithm (AV) and the methods to decode of turbo codes (TC) for long codes are too complex. The complexity of optimal AV grows exponentially with an constructive code length *K*, therefore practically such method is usually used to decode codes with $K \le 9$ showing low efficiency [2]. The complexity of decoders for TC is determined by the complexity of constituting codes decoders and the number of decoding iterations. It also turns out to be too high to be used in high-speed communication systems [2]. One of the most efficient and simple to be realized decoders is multithreshold decoder (MTD) of self-orthogonal codes (SOC) [3..8]. The complexity of MTD is proportional to coding distance of the codes used and the number of decoding iterations. Considering typical encoder and multithreshold decoder parameters this complexity is hundred times less than the complexity of decoders for turbo codes [2]. At the same time standard MTD provides a little less coding gain than TC decoders with same code parameters. To increase MTD efficiency the approaches based on concatenation can be used [9, 10]. But in this case it is necessary to make changes in communication systems of both transmitter and receiver which is not always possible. [11] offers a method to improve SOC decoding efficiency due to decoding of received data by several MTDs having different settings with consequent forming of decoding result on the basis of majority votes. Such decoder was called multilevel MTD [11]. It should be noted that such decoding scheme requires additional study and its efficiency can be improved.

Purpose of the work is study and improvement of self-orthogonal codes decoding efficiency with the help of multilevel MTD.

II. Multithreshold decoding

The multithreshold decoders are used for decoding of block and convolutional self-orthogonal codes. The basic principles of MTD is illustrated with scheme of fig. 1. In the fig. 1 the scheme of MTD for a block SOC of rate 1/2 with generation polynomial $g(x)=1+x^1+x^4+x^6$ is shown [3]. Note the MTD consists of shift registers, modulo 2 adders and threshold element (TE). The TE calculates sum of inputs and compares the sum with threshold. So MTD is very simple for implementation decoder providing high decoding rate.

For a binary symmetrical channel the TE of MTD for an information symbol i_j calculates the usual sum of the syndrome component s_{j_k}, containing as additives the error e_j in the decoding symbol i_j (i.e. we need to find the sum of checks $s_{j_k} \in \{S_j\}$, where $\{S_j\}$ is a set of checks related to the component e_j, corresponding to a symbol i_j) and the symbol d_j (difference vector component), which is also related to decoded symbol i_j:

$$L_j = \sum_{s_{jk} \in \{S_j\}} s_{j_k} + d_j \,. \qquad (1)$$

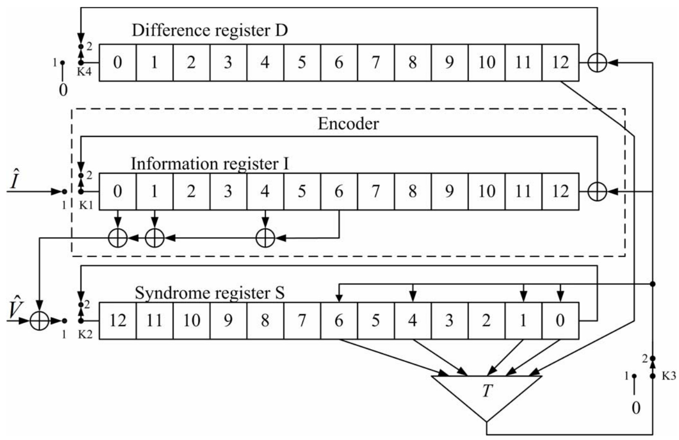

Fig. 1. An example of MTD for a block code

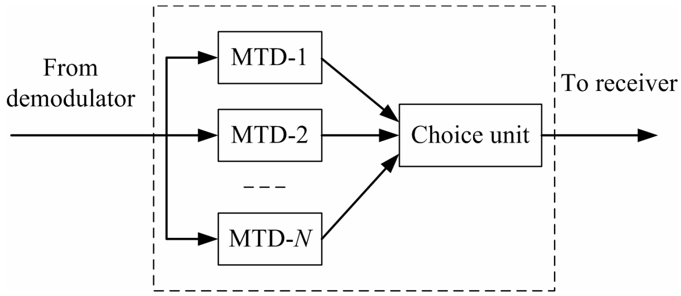

Fig. 2. An example of parallel multilevel MTD

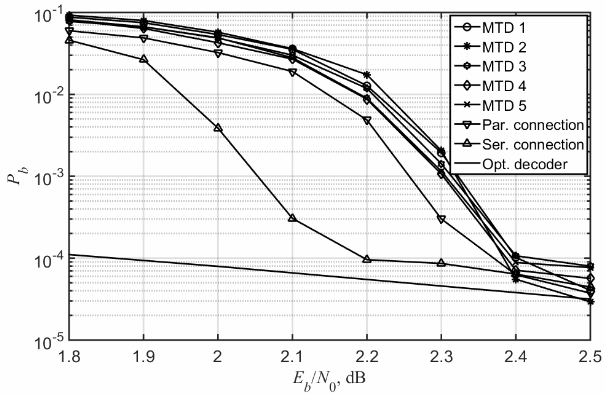

Fig. 3. The performance of parallel multilevel MTD

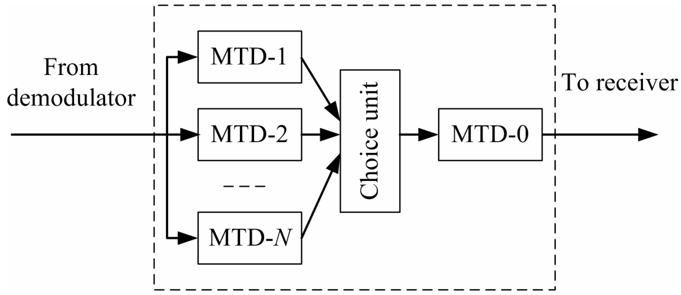

Fig. 4. An example of serial-parallel multilevel MTD

All J=d–1 checks (d is the code distance), and also i_j and d_j are inverted for $L_j > T$ and remain unchanged for $L_j \le T$, where T is the threshold value usually equal to a half of all addends in (1).

For an AWGN channel addends in (1) are summed with weights depending on the reliability of received symbols:

$$L_j = \sum_{s_{j_k} \in \{S_j\}} (2s_{j_k} - 1)w_{j_k} + (2d_j - 1)w_j. \qquad (2)$$

It's known [3] the MTD with each change of the decoded symbols finds new codeword is closer to the optimum decoder decision as common weight of syndrome and difference vectors is lower. It is crucial that MTD complexity should remain the same as for customary threshold decoder, namely linear, i.e. theoretically the lowest possible.

Let's note most important feature characterizing multithreshold algorithm. In case of binary codes we can't claim that MTD solution improvement during multiple decoding attempts will take place till optimum decoder decision is achieved. In fact both in block and in convolutional codes it's possible to meet such error configurations which cannot be corrected in MTD, but some of them can be corrected in optimum decoder. That's why the main way to increase MTD efficiency is to search codes where these noncorrected error configurations are quite rare even in high level of noise. The questions to choose such codes are considered in detail in [3].

III. Parallel multilevel MTD

Let us consider first the multilevel MTD version using only parallel connection of several MTD (fig. 2). In such decoder soft solutions of demodulator are sent in parallel on N constituent MTDs. All N of decoders make their own decisions concerning decodable symbols resulting in N decoded messages entering choice unit. This unit forms the decoding result of each bit of the message on the basis of majority votes.

To get most efficiency it is desirable that all N of MTDs have maximum similar efficiency, besides, decoders parameters must be different enough to provide different combinations of errors left at the output of decoders.

Let us consider the performance of the given decoder in the channel with additive white Gaussian noise (AWGN) with using binary phase-shift keying (BPSK) and 16-level quantization at the output of demodulator.

Fig. 3 shows the simulation results of given decoder for block SOC with code rate R=2/4, minimum code distance d=9 and the code block length n=20748. Five MTDs with different weights and thresholds were used at simulation, the number of iterations was I=15. The BER performance is these MTD is shown by the curves «MTD 1»…«MTD 5». The curves show that all five decoders have similar efficiency. The multilevel MTD (curve «Par. connection») provides additional 0,1 dB coding gain in comparison with standard MTD. It should be noted that in the area of low noises the gain can't be received as in this area each MTD operates as an optimum decoder for the code used (curve «Opt. decoder»), the efficiency of which can't be improved. It can also be observed that the complexity of parallel circuit in comparison with the complexity of standard MTD increases several times, but does not exceed the complexity of other efficient algorithms.

IV. Serial-parallel multilevel MTD

Let us look at the performance of multilevel MTD where after the choice unit one additional external MTD is used. Such decoder is presented in fig. 4. In this case with help of external MTD decoding of the message received after the choice unit is made. This allows to increase more the efficiency of decoding.

The simulation results for serial-parallel multilevel MTD containing six MTD for the same conditions are presented on fig. 3 by the curve «Ser. connection». As it can be seen, while using additional MTD after the choice unit the coding gain up to 0,2 dB in comparison to parallel connection can be received. At the same time the complexity of the decoder increases only by 20 %.

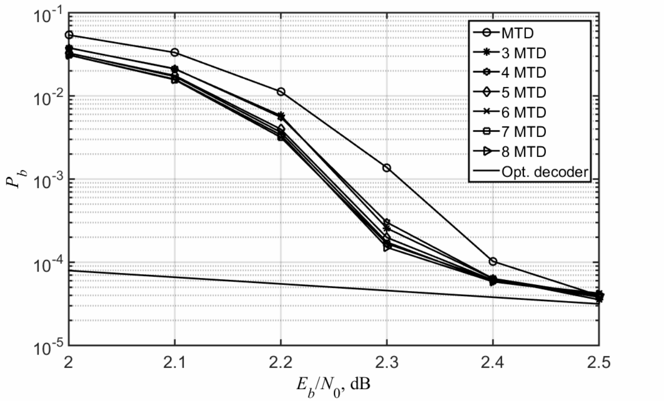

Fig. 5. The performance of parallel multilevel MTD

V. THE PERFORMANCE OF MULTILEVEL MTD WITH DIFFERENT NUMBER OF DECODERS

The BER performance over AWGN channel with BPSK modulation for parallel multilevel MTD with three to eight constituting MTD is presented in fig. 5. As shown in the figure using three and four constituting decoders gives us the gain up to 0,05 dB, five decoders usage leads to the gain up to 0,1 dB. While using five to eight decoders no increase can be seen. Consequently, the recommended number of decoders for this circuit is five. The complexity of this multilevel MTD in comparison with standard MTD is increased proportionally to the number of constituting decoders.

Fig. 6 shows the simulation results for the serial-parallel multilevel MTD with three to eight constituting MTD on the first step before the choice unit. As it can be seen, using three and four decoders on the first step leads to the gain 0,2 dB. The increase of constituting decoders up to five allows to provide the coding gain about 0,3 dB. The increase in constituting decoder number can be further continued but it increases the complexity of decoder while the gain turns out to be little.

Consequently, serial-parallel multilevel MTD provides 0,2 dB better characteristics in comparison with parallel circuit and 0,3 dB better ones in comparison with standard MTD. The number of decoders influences the characteristics of serial-parallel the same way as the characteristics of parallel circuit. It is recommended to use five constituting decoders before the choice unit which gives good result while providing enough simplicity of decoder implementation.

VI. THE PERFORMANCE OF MULTILEVEL MTD WITH DIFFERENT CHOICE UNIT OPERATING ALGORITHM

In [11] the BER performance of serial-parallel multilevel MTD with the majority approach to determine message bit value being used by the choice unit has been analyzed. At such approach the choice unit uses only information bits received after constituting MTD and chooses each bit on the basis of voice majority. The basic step of this choice means

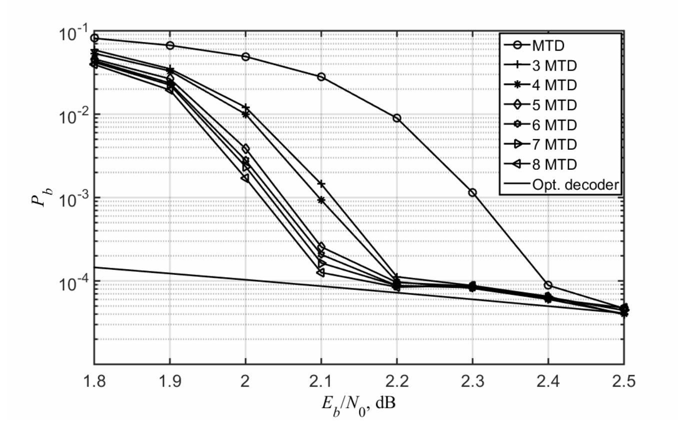

Fig. 6. The performance of serial-parallel multilevel MTD

that for each analyzed symbol u_i the likelihood function L_i, depending on $u_{i,k}$ symbols, received from N internal decoders is calculated

$$L_i = \sum_{k=1}^{N} (2u_{i,k} - 1) . \qquad (1)$$

If $L_i>0$, then u_i symbol equals 1, if not then $u_i=0$.

It should be noted several alternative algorithms to determine such a decision, using the information about constituting MTD decisions reliability, can be offered. The algorithms for choice unit as the choice of the signal with maximum reliability and majority choice considering weight of decoded symbol are offered and studied below.

First, let us examine the first algorithm when **the choice unit determines information bits according to the symbol with maximum reliability.** The reliability of decoded symbol is understood here as sum values for this symbol on threshold element of constituting MTD after the symbol having been decoded. Let us designate $w_{i,k}$ as the value of the reliability of i-th information symbol received from k-th constituting MTD. To estimate the information symbol u_i N values of reliability $w_{i,k}$ are compared to choose the value u_i, which is equal to symbol from the constituting decoders with highest reliability value:

$$u_i = u_{i,m} \text{, where } m = \arg\max_k (w_{i,k}) . \qquad (2)$$

The BER performance of parallel and serial-parallel multilevel MTD while choosing information symbol of maximum reliability for block SOC with code rate R = 2/4, code distance d = 9, code length n = 20748 having five parallel constituting MTD with I=15 decoding iterations is presented in fig. 7 by the curves «Par. max.» and «Ser. max.». These graphs are received for the AWGN channel with using BPSK modulation and 16 level quantization of demodulator decision.

To compare "MTD" curve in fig. 7 shows the performance of standard MTD for the same code, "Opt. decoder" curve shows the BER of an optimum decoder for the SOC used, and "Par. hard" with "Ser. hard" curves show the characteristics of parallel and serial-parallel circuits while using majority operating algorithm of choice unit. The comparison of these curves shows that choosing information bit according to maximum reliability in the parallel multilevel MTD gives no gain in comparison with majority voting, but the gain up to

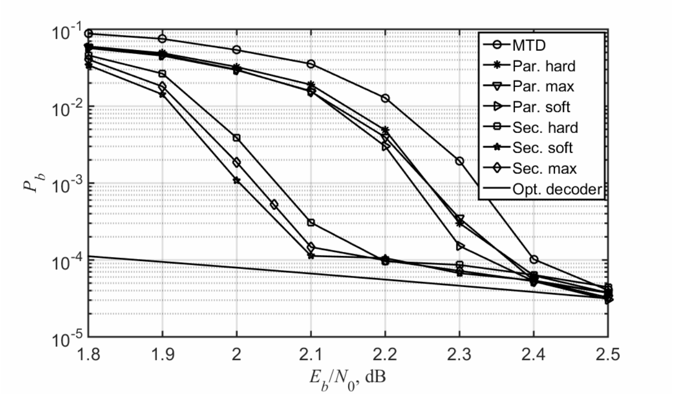

Fig. 8. The performance of multilevel MTD with different information symbol choice algorithms

0,05 dB can be observed in the serial-parallel multilevel MTD.

The second algorithm is based on **majority choice of information bit taking into consideration reliabilities of corresponding bit formed by all constituting MTD.** In such case choice unit receives *N* MTD decisions $u_{i,k}$ regarding *i*-th information symbol and their reliability $w_{i,k}$. For each u_i bit the choice unit calculates likelihood function L_i

$$L_i = \sum_{k=1}^{N} (2u_{i,k} - 1) w_{i,k} \ . \qquad (3)$$

If $L_i>0$, then u_i symbol takes the value 1, otherwise $u_i=0$.

The BER performance of parallel and serial parallel multilevel MTD with given algorithm of choice unit operation for the same conditions as were before are represented in fig. 7 by the curves "Par. soft" and "Ser. soft". Note that the usage of majority choice of information bit considering reliabilities formed by constituting decoders leads to better (up to 0,05 dB) coding gain.

VII. The performance of multilevel MTD for the code with larger code distance

Communication systems requiring low error rate demand the code with larger code distance. Let us consider the performance of the multilevel MTD with I=13 decoding iterations which use five constituting MTD for SOC with code rate R = 8/16, code length n = 63472 bits and code distance d=17 and in the same conditions as earlier. The performance is represented in fig. 8 by the curves named the same as in fig. 7. It should be noted that the serial-parallel multilevel MTD gives the gain about 0,3 dB even for codes with large code distance.

VIII. Conclusion

The main advantage of MTD algorithms alongside with high efficiency is the possibility of their extremely high performance both in their software and hardware implementations. This fact allows to combine several MTD in one decoding scheme to get higher coding gain keeping the

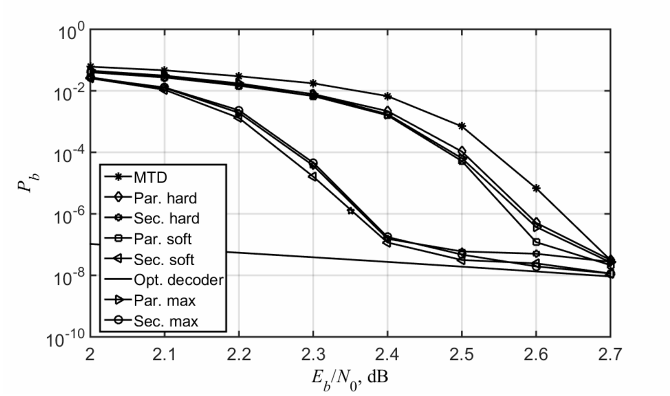

Fig. 7. The performance of multilevel MTD during different operating choice algorithms of information bit for the code with d = 17

simplicity of implementation in comparison with other methods of error correction.

The algorithms of choice unit operation offered in the article which determine the values of information symbol for external multithreshold decoder of multilevel MTD allow to improve performance of decoder by a few tenths of dB practically without the increase of implementation complexity.

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References

[1] Zubarev Yu.B.,Ovechkin G.V. Error-correcting coding in digital data communication systems // Electrosvyaz. M., 2008. №12. P. 2-11.

[2] Zolotarev V.V., Ovechkin G.V. The complexity to implement efficient methods of error-correcting codes // 6 international conference and workshop «Digital signal processing and its application». M.: 2004. Volume 1. P. 220-221.

[3] Zolotarev V.V., Zubarev Yu.B.,Ovechkin G.V. Multilevel decoders and optimization coding theory. – M.: Hot line – Telecom, 2012. 239 ps.

[4] Zolotarev V, Ovechkin G., Satybaldina D., Tashatov N., Adamova A., Mishin V. Efficiency multithreshold decoders for self-orthogonal block codes for optical channels // International Journal of Circuits, Systems and Signal Processing. ISSN 1998-4464. – 2014. Vol.8. pp.487–495.

[5] Ovechkin G., Zolotarev V., Ovechkin P., Satibaldina D., Tashatov N. The Performance of Concatenated Schemes Based on Non-binary Multithreshold Decoders // Advances in Systems Science. Springer International Publishing, 2014. Vol.240. P.251-259.

[6] Ovechkin G.V., Zolotarev V.V., Averin S.V. Algorithm of multithreshold decoding for self-orthogonal codes over Gaussian channels – 11-th ISCTA’09, July, UK, Ambleside, 2009.

[7] M.A. Ullah, K. Okada, H. Ogivara. Multi-Stage Threshold Decoding for Self-Orthogonal Convolutional Codes. IEICE Trans. Fundamentals, Vol.E93-A, No.11, pp. 1932 -1941, Nov. 2010.

[8] M.A. Ullah, R. Omura, T. Sato, H. Ogivara. Multi-Stage Threshold Decoding for High Rate Convolutional Codes for Optical Communications. AICT 2011: The Seventh Advanced international Conference on Telecommunications, pp. 87-93.

[9] Grinchenko N.N., Zolotarev V.V., Ovechkin G.V., Ovechkin P.V. Multithreshold decoding in the channels with multipositional modulation // Vestnik of Ryazan state radioengineering university, 2006. Issue 19, P. 179-182.

[10] Ovechkin G.V. Methods to improve the efficiency of multithreshold decoder of self-orthogonal codes // Vestnik of RSREA 2004. P. 54-58.

[11] Dmitriyeva T.A. Development of multilevel multithreshold decoding unit – Vestnik of Ryazan state radioengineering university. Issue 22. Ryazan, 2007. P. 73-78.