Effective multithreshold decoder for optical and other data transmission systems

Valery Zolotarev, Gennady Ovechkin, Dina Satybaldina, Nurlan Tashatov, Aigul Adamova, and Vitaly Mishin

Abstract—Multithreshold decoding algorithms for self-orthogonal error-correcting codes are considered. Recent advances in the field of coding and decoding of binary codes, which are used in various high-speed satellite channels, are presented, as well as new opportunities decoders of the same type for use in optical communications. Brief comparison of characteristics of multithreshold decoder and other decoders (Viterbi and turbo decoders, decoder for low density codes) are considered.

Keywords—error-correcting codes, iterative decoding multithreshold decoding, optic communications.

I. Introduction

Currently various digital communication systems are applied for the exchange of information. Such systems are used for data transmission with wired and wireless communication channels, in which information can be distorted under the influence of various kinds of interference. It is unacceptable for many applications. Therefore, error-correcting coding tools are used in any digital transmission, and its using reduces the proportion of uncorrected errors to an acceptable.

Absolute necessity to use error-correcting coding has been elucidated for information channels for many decades ago. Use of codes reduces the need in signal power per 10 times. This is extremely important in many cases the use of digital radio. This is especially important on board the spacecraft, when the increase in actual physical transmitter power is technically impossible. Currently develops optic communication lines (OCL) that provide transmission of large amounts of data at high speed around a hundred Gbit/s. Error-correcting coding is used to improve reliability of data transmission systems, the use of which allows to increase efficiency channel usage. The main requirement is to ensure for the schemes of coding and subsequent decoding OCL with a very high reliability (the probability of error of about 10^{-17}) extremely fast decoding. Therefore, OCL can be applied only with the fastest decoders.

Fastest decoders should only consist of a large number of the fastest microelectronic elements - large blocks of memory or long shift registers. They should not contain long chains of feedbacks, which greatly reduces the rate of advance data on such registers. Results in [1, 2] showed that the most suitable for high-speed systems according to these criteria are multithreshold decoders (MTD) for self-orthogonal codes [3, 4, 5]. For MTD shown that they allow almost optimal (i.e., as well as then iterative exponentially complex code length methods) to decode even very long codes with linear complexity of implementation, that demonstrating good correction capability.

In present paper some new important MTD properties are discussed. This article reviews operation principles of multithreshold decoders, compares their efficiency with efficiency of other error correction methods and presents possibilities of the MTD for high-speed codes, suitable for use in the OCL.

II. Multithreshold Decoding

Let's describe operating principles of MTD for SOC decoding [3, 4, 5]. For implementation of operation of encoding SOC it is possible to use the elementary diagrams constructed on the basis of shift registers. The example of the diagram of the coder block SOC, set by an ancestor polynomial $g(x) = 1+x +x^4 + x^6$, is shown in a Fig.1. This code is characterized by parameters of code length, length of information sequence, code speed and the minimum code distance of $n=26, k=13, R=1/2, d=5$, respectively. The similar diagram is used for encoding convolution SOC.

Let's describe the principles of operation of the encoder on the example provided by scheme. Check bits generates in the encoder during operation in accordance with the following algorithm:

1. Before starting to encode code block key K is in state 1.

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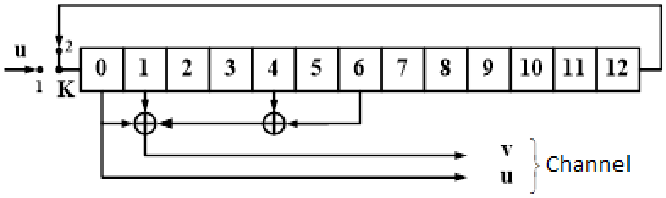

Fig. 1 Encoder block SOC with R = 1/2, d = 5 and n = 26

2. Information vector $\mathbf{u} = (u_0, u_1, \ldots, u_{12})$ applied one character input shift register. As a result, information symbol u_0 is located in cell 12, u_1 - the cell 11, etc.

3. Key K is transferred to state 2.

4. For j from 0 to 12 to perform cyclic shift register, and then calculates the j-th checking bit v_j:

$$v_j = \sum_{k=1}^{4} u_{(j-g_k) \bmod 13} \quad . \tag{1}$$

As a result of the algorithm generated a checking vector $v = (v_0, v_1, \ldots, v_{12})$, which, together with an information vector defines the code word $c = (u, v)$, which is transmitted through the channel.

Let's describe the principle multithreshold decoding of SOK. In a situation, where the decoder after transmission of a binary symmetric channel (BSC) rather than a distorted codeword noises message $y = (u', v')$ of length n. First calculated syndrome $\boldsymbol{s} = \mathbf{H}\boldsymbol{y}$ (here H - check matrix code) of the received message, and for each information symbol u_j, $1 \le j \le k$, stands set $\{s_p\}$ syndrome elements with numbers $\{p\}$, called checks relative to the character u_j and containing error e_j in this symbol.

First, as in the usual threshold decoder is calculated syndrome $s = Hy$ (here H - check matrix CSOC) of the received message, and each information symbol u_j, $1 \le j \le k$, find the set of elements $\{s_p\}$ syndrome with numbers $\{p\}$ called checks against symbol u_j and containing, as an error term e_j in this symbol.

In addition to the threshold decoder in MTD injected binary vector d of length k, called the difference, initially filled with zeros. The basic step is to decode that for arbitrarily chosen symbol u_j computed likelihood function L_j, independent of its related inspections and j-th element of vector d:

$$L_j = \sum_{p \in \Theta_j} S_p + d_j \tag{2}$$

where d_j - a symbol of the difference vector, related to decoded symbol u_j (0 or 1); S_p - p-th element of the syndrome vector, which is part of a number of checks regarding decoded symbol u_j; Θ_j - a set of of numbers of checks, controlling the j-th information symbol.

The example of MTD implementation for encoder from Fig. 1 is given in Fig. 2.

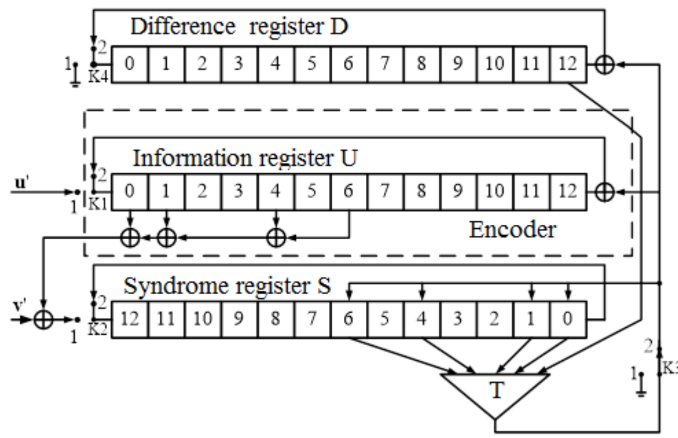

Fig. 2 Multithreshold decoder for SOC

MTD can be easily modified as the normal threshold decoder for adding checks in (2) with certain coefficients. Output bits define reliability for the decision made when dealing with multiple levels quantized soft modem solutions. Using of soft decisions of the demodulator can achieve 1.4 ... 1.7 dB better results than using hard decisions of the demodulator. The expression (2) to calculate the likelihood function takes the form L_j

$$L_i = \sum_{p \in \Theta_j} (2s_p - 1)w_p + (2d_j - 1)w_j \tag{3}$$

where $\{w_p\}$– factors reflecting the reliability checks $\{s_p\}$; w_j – factor reflecting the reliability of the received symbol u_j. Logarithm of the likelihood ratio can be used as the grade received from the channel symbols

$$w_j = \ln \frac{P(u_j = 1 | r_j)}{P(u_j = 0 | r_j)} \tag{3}$$

where r_j – symbol received from the channel corresponding to the transmitted information symbols u_j. Values w_j. can be quantized into several levels to simplify the calculations.

III. Efficiency of the Multithreshold Decoder

Let's compare characteristics of MTD and other non-binary error correction methods in channels the additive white gaussian noise (AWGN) and a binary phase modulation (FM2) for the binary codes with a code rate R = 1/2. Theoretically, decoder can work with these parameters channel and codes when the signal/noise ratio equal of 0.2 dB (curve "C = ½" in Fig. 3).

Convolutional codes have found most widespread practical use in actual communication systems. Viterbi algorithm [6] and various concatenated codes is often used to decode them.

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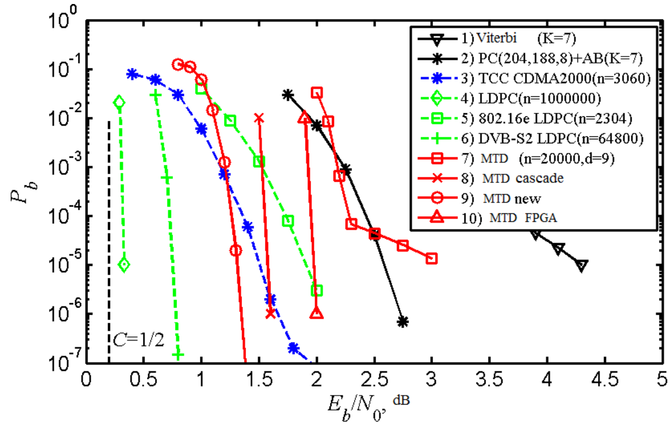

Fig. 3 Performance of error-correcting codes with R = 1/2 over AWGN channel and FM2

These methods emerged and developed in the 70s - 80s of the last century. Turbo [7, 8] and low density codes [9] are actively developed in recent times by foreign experts, the effectiveness of which is very high. For example, methods for decoding turbo codes recommended standard CDMA2000, provide characteristics represented by a curve «3) TCC CDMA2000(n=3600)».

Small probability of error decoding can be achieved by low-density codes of length a million bits when working less than 0.1 dB capacity a Gaussian channel (LDPC(n=1000000)). Efficiency of decoders low density codes of shorter length is shown in Figure by curved lines «5) 802.16e LDPC(n=2304)» and «6) DVB-S2 LDPC(n=64800)». Unfortunately, all of these methods when working in a big noise still have a very large implementation complexity, making it difficult to practical use in high-speed data transmission and storage. Efficiency of MTD is presented for code with length of 20,000 bits, a code distance d = 9 and code rate R = 1/2 in Fig. 3 «Curve 7) MTD (n = 20000, d = 9)». MTD perform only a quick simple addition and comparison of integers, which makes them very attractive for use in existing and newly developed high-speed digital data transmission systems.

Developers are constantly looking for ways to increase its efficiency, despite the good correction capability provided by the original MTD algorithm.

One way to approach the area of effective MTD work up your bandwidth is code selection, the least prone to error propagation (EP) decoding [3, 4, 5]. This property is reflected in the fact that after the decoder at work makes a mistake, the error probability in the following symbols increases significantly. In [3, 4, 5] proposed an approach to assess the susceptibility of code and build EP codes with minimum EP. Codes are necessary to obtain the best performance, in which there are multiple branches of information and verification. Example encoder such code is shown in Fig. 4 containing two information and two checking branches.

When using code such a structure can achieve significant reduction in the breeding of errors by reducing the number of common errors involved in decoding the various bits of information [3, 4, 5].

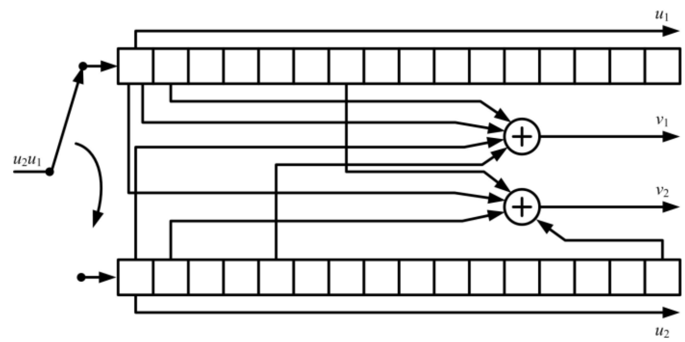

Fig. 4 Encoder block SOC with two information and two checking branches

When using code such a structure can achieve significant reduction in the breeding of errors by reducing the number of common errors involved in decoding the various bits of information [3, 4, 5]. In [10], the authors show that only by a proper choice of code and optimization of its structure without complicating decoding scheme can get additional energy gain of the order of 1 .. 1.5 dB.

The next area is the work under these extremely efficient and extremely simple algorithms associated with the development of concatenated coding schemes. Cascading should only be with very simple codes to the overall complexity of the scheme has not increased. Focuses on this approach concatenated codes used in the MTD, with parity codes, Hamming codes and short self-orthogonal codes [11]. Analytical calculations and computer simulation results show, that application of such schemes allows you to bring the effective area of MTD bandwidth channel 1 .. 2 dB and reduce the probability of error decoding for 2 .. 5 orders of magnitude without a significant complication of the decoding scheme. This scheme allows to provide efficacy comparable with the efficiency of the best methods of error correction. The complexity of the cascade scheme decoder is very small. As a result of this concatenated MTD easy to implement as a conventional MTD decoder for speeds of 500 Mbit / s or even higher.

Additional improvement in decoding performance self-orthogonal codes possible with non-significant complication decoding algorithm [12]. Features presented one such decoder in Fig. 3 with curve "9) MPDnew". It illustrates the very high energy efficiency of the proposed algorithm at a distance of only 1.1 dB of channel capacity. The absolute majority of other error correction algorithms are in such high noise extremely difficult. Provided characteristics comparable or even better than many well-known characteristics of turbo decoders and LDPC codes.

The analysis of implementation complexity shown that MTD performs decoding ten, fifty or more times fewer operations than other comparable methods of error correction efficiency [3, 4, 5]. MTD algorithm can be recognized as one of the main methods of error correction for most of today's high-speed data transmission systems with the maximum possible energy levels and win speed reaches several Gb/s.

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MTD is possible for full parallelization of operations in its hardware implementation [13].

One of the latest implementations of the MTD was developed in IKI on the base of FPGA (Altera Stratix EP1S20) [13]. This project has shown that we can always get good energy characteristics of coding at a high level of noise on the data rate in excess of 1 Gbit/s at extremely low hardware complexity. This is extremely valuable for remote sensing (RS) and for all new systems of micro-and nano-satellites. Example characteristics provided by this FPGA is shown in Fig. 3 with curve 10.

The major advantage offered by MTD algorithms along with high efficiency is the possibility of extremely high performance hardware implementation. Since these algorithms allow their full parallelization, it allows decoding MTD in rate that matches the speed of shift registers (for the high-speed circuit design elements!) in the selected element base. Currently no known other types of algorithms that would at least partially possessed similar properties. These features high speed MTD work you can always save at virtually any modifications and improvements MTD methods known to date.

MTD methods are truly unique algorithms capable of providing efficient decoding at high noise level. They perform a very small number of transactions and the highest levels of reliability of storing digital information and its processing speed in very large-scale databases, optical disks, etc. In all these cases very limited resources are used, such as simple microprocessors or the cheapest FPGA, which determines the ease and efficiency of the new methods of error-correcting the coding.

IV. MTD APPLICATION TO IMPROVE THE RELIABILITY OF DATA TRANSMISSION IN THE OCL

Let's consider the possibilities MTD used in conjunction with high-speed codes, for example, suitable for applying in the OCL. Fig. 5 shows the characteristics of various MTD for convolutional codes with a code rate R = 4/5 in a Gaussian channel. Curve 1J shows the possibilities of the use of fairly complex MTD decoder of Japanese specialists [1,2]. These schemes present the feedback data transfer, markedly reducing speed of the circuit that these networks would be more high speed. Curve 2Dec given for MTD decoder using code, the lower bound optimum decoding which corresponds to the curve 2Opt.

Simulation results shown that increasing of the ratio Eb/N0 signal/noise values we seen decrease of error probability Pb (e) for MTD with such codes. MTD characteristics reached a level of optimum decoder at E_b/N_0=3,7 dB or less. The results of these experiments suggest that the code with MTD will reach the optimum decoder level. It is determined by the lower boundary 2Opt with a noise level of 3.7 dB. Achieving a level of optimum decoding can be taken for granted, at least when

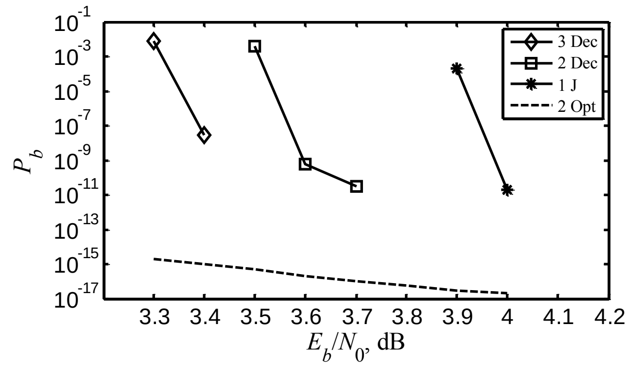

Fig. 5 Efficiency MTD for codes with a code rate R = 4/5 over Gaussian channel

E_b/N_0=3,8 dB.

The curve 3Dec corresponds to the option MTD application when not required to achieve conventional coding methods noncascade very small error probability at the output of the decoding apparatus. Curves 2Dec and 3Dec match MTD work at significantly greater level of noise than the decoder, the characteristics of which are given in the curve 1J. MTD represented by curve 3Dec, requires about 0.5 million code symbols delay decision in 25 iterations of decoding convolutional code.

It is understood that the ability to operate at high noise level allows the use of all the algorithms with different modifications in MTD and various types of circuits cascaded. All results of the three options MTD shown in Fig. 5, in the case of cascade schemes error correction will be, of course, improved. However, even when the characteristics of the second code are initially noncascade better than in concatenated coding schemes obtained in [2, 14]. Improvement of parameters third code will be particularly noticeable at staging because it operates at a higher noise level than the two previous. But when cascading all types always have to take additional measures in order to not greatly reduce the processing speed, as this violates the principle of instant correction of errors in the MTD as it moves through shift registers decoder.

V. CONCLUSIONS

It is shown that a fundamentally new level of performance and processing speed compared with absolutely all known methods of error correction can be achieved by using different types of MTD algorithms. MTD algorithms allow us to solve the problem to ensure high reliability of data transmission without any additional modification of these algorithms. Their use is equally simple and effective at the hardware and software implementation.

Great deal of additional information on multithreshold decoders can be found on websites [14].

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