



**ITMAR-15**

**Multi level Multithreshold Decoding of Self-Orthogonal Codes for High-Speed Communications**

Nurlan Tashatov<sup>1\*</sup>, Dina Satybaldina<sup>2</sup>, Natalya Grinchenko<sup>3</sup>, Van Toan Cao<sup>4</sup>, Gennady Ovechkin<sup>5</sup>

<sup>1,2</sup>*L.N. Gumilyov Eurasian National University, Republic of Kazakhstan,*  
<sup>3,4,5</sup>*Ryazan State Radio Engineering University, Russian Federation*

**Abstract**

Multilevel multithreshold decoders (MMTD) for self-orthogonal error-correcting codes are considered. Recent advances in the field of error-correcting coding, which are used in various high-speed communication channels, are presented, as well as new opportunities decoders of the same type for use in optical networks. The SER performance of MMTD is shown to be close to the results provided by optimum total search methods. The performance of the concatenated coding schemes (a parallel and series-parallel concatenation of several MMTD) is presented. Recommendation on selecting the best algorithm for decision block and the best parameters for decoders is given. New methods for MMTD performance improving at the expense of better usage of decoded bits reliability with decision block are proposed.

© 2015 The Authors. Published by Global Illuminators. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

Peer-review under responsibility of the Scientific & Review committee of ITMAR- 2015.

*Keywords*— Communications, Error-Correcting Codes, Iterative Decoding, Multithreshold Decoder, Multilevel Multithreshold Decoder.

**Introduction**

Coding theory is the branch of mathematics concerned with transmitting data across noisy channels and recovering the message (Richardson & Urbanke, 2008). The value of error-correcting codes for information transmission, both on Earth and from space, was immediately apparent, and a wide variety of codes were constructed which achieved both economy of transmission and error-correction capacity.

Such error-correction coding systems as decoders of Reed-Solomon codes (Hanho Lee, 2005), convolutional codes with decoding by Viterbi algorithm (AV) (Viterbi, 1967; Cristea, 2010), turbo codes (TC) (Berrou & Glavieux & Thitimajshima P, 1993; Wilde & Min-Hsiu Hsieh & Babar, 2014; Breddermann & Vary (2014).) and low-density parity-check codes (LDPC) (Richardson & Shokrollahi & Urbanke, 2001; Sharifi & Tanc & Duman, 2015) are widely used in digital communications.

Comparison of the decoding algorithms shows that the most efficient algorithms, such as AV and TC decoders in case of the long codes have a high complexity of implementation (Zolotaryov, 2006).

The complexity of optimal AV grows exponentially with length code  $K$ , and therefore it is usually used for decoding codes with  $K \leq 9$ , possessing low efficiency (Robertson & Villebrun & Höher, 1995). Complexity of TC decoder depends on the complexity of decoding methods of the component codes and the number of decoding iterations. It also proves to be too great for them to use in the high-speed communication systems (Höher & Robertson & Villebrun, 1997).

\*All correspondence related to this article should be directed to Dina Satybaldina, Gumilyov Eurasian National University, Republic of Kazakhstan  
Email: [satybaldina\\_dzh@enu.kz](mailto:satybaldina_dzh@enu.kz)

© 2015 The Authors. Published by Global Illuminators. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

Peer-review under responsibility of the Scientific & Review committee of ITMAR-2015.

As efficient error-correction method a multithreshold decoding (MTD) for self-orthogonal codes was offered in (Zolotaryov, 1973). The method is possesses property of aspiration to the solution of the optimum decoder with preserving linear complexity of decoder implementation at variation of length code  $K$  (Zolotaryov, 2003). MTD is characterized with a very small number of operations, soft versions of these decoders can correct in channels with high enough noise level streams of the data in high-speed communication systems (Zolotarev & Ovechkin, 2010).

MTD complexity (in case of the typical parameters of the encoder and decoder) is hundreds of times less complexity TC decoders, but MTD performance is usually slightly less than performance of TC decoders (Zolotaryov, 2006).

To increase the effectiveness MTD we were used the approaches based on concatenation (Ovechkin & Satyaldina & Tashatov, 2014; Ovechkin & Zolotarev & Satyaldina, 2014).

*Objective of the Study*

The current study aimed at investigating and improving the efficiency of self-orthogonal code decoding using a multilevel multithreshold decoding (MMTD) (a parallel and series-parallel concatenation of several multithreshold decoders).

Scheme of the Parallel Concatenation of Several Multithreshold Decoders

In this scheme, demodulator's soft decisions are sent in parallel on the  $N$  components MTD (as seen at Figure 1).  $N$  decoded messages from  $N$  decoders are sent to the decision block. The decision block forms a decision as result of decoding each bit on the majority principle.

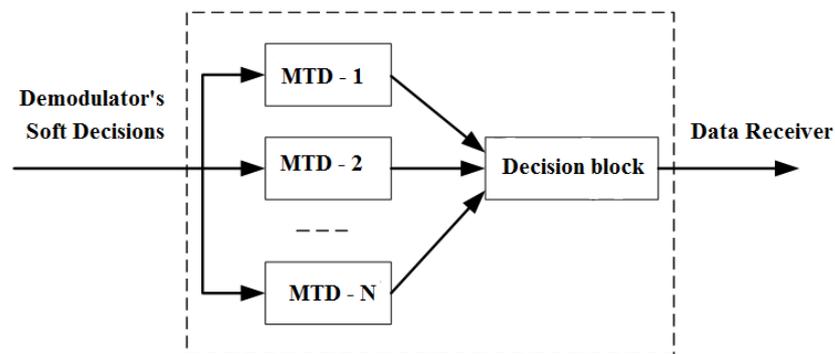


Figure1: A parallel concatenation of several MTD

Let's consider the characteristics of multilevel multithreshold decoding scheme in channels the Additive White Gaussian Noise (AWGN) and a binary phase modulation (BPSK) for the convolutional codes (a code rate  $R = 2/4$ , a length code  $K=9$  and a length block  $n = 20748$ ) and 16 levels of quantization solutions at the demodulator output (Figure 2).

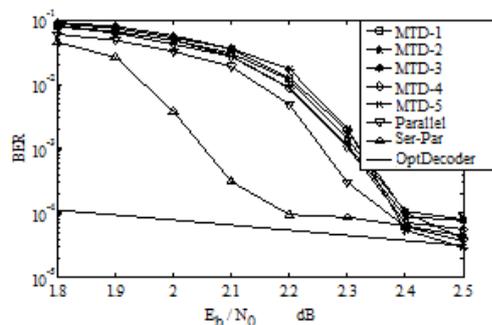


Figure 2: Performance of the parallel concatenation of several MTD over AWGN channel BPSK

Five different decoders (MTD 1-5) with different parameters of the weights and thresholds were used in the simulation, the number of iterations  $I = 15$ . Efficiency of the decoders are shown in Figure 2 by curved lines « MTD 1-5». It is seen that all five decoders have comparable efficacy. Multilevel MTD (curve "Parallel" at Figure 2) allows to bring the effective border of the decoder in the high noise bandwidth to approximately 0.1 dB compared with a non-multilevel MTD. It is seen that a coding gain can't be increased at low noise. In this area, each MTD works almost like an optimal decoder for used code (curve "OptDecoder" at Figure 2), the effectiveness of which can't be improved.

#### Scheme of the Serial - Parallel Concatenation of Several Multithreshold Decoders

Let's consider the effectiveness of multilevel multithreshold decoding in which one decoder is additionally used after the decision block. Scheme of the serial - parallel concatenation of several multithreshold decoders is presented at Figure 3.

In this case, the bit sequence from decision block is further encoded by using an external decoder. This change the scheme allows to increase decoding efficiency (curve "Ser-Par" at Figure 2). We can see that additional coding gain to 0.2 dB compared to the parallel scheme. In this case the complexity of the MMTD implementation increases only by 20%.

Effectiveness of the proposed schemes was investigated by increasing the encoding parameters. Five multithreshold decoders were used for convolutional codes decoding with code rate  $R = 8/16$ , a length code  $K=17$  and a length block  $n = 63472$ , the number of iterations  $I = 13$ . Note that in this case serial - parallel concatenation of several multithreshold decoders gives a gain of 0.3 dB.

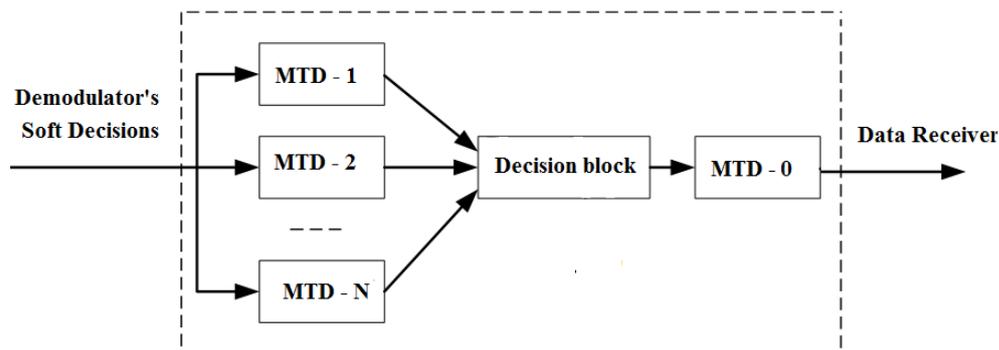


Figure 3: A serial-parallel concatenation of several MTD

#### Conclusion

It is shown that a fundamentally new level of performance and processing speed compared with absolutely all known methods of error correction can be achieved by using different types of MTD algorithms.

Schemes of multilevel multithreshold decoding allow us to solve the problem to ensure high reliability of data transmission without any additional modification of these algorithms. Their use is equally simple and effective at the hardware and software implementation.

Researches of MTD algorithms were supported by the Russian fund of fundamental researches (grant No. №14-07-00824), the grant of the President of the Russian Federation (grant МД-639.2014.9) and the Science Committee of Ministry of Education and Science of the Kazakhstan Republic.

## References

- Berrou C., Glavieux A. Thitimajshima P. (1993). Near Shannon Limit Error-Correcting Coding and Decoding, Turbo Codes. *Proc. of the Intern. Conf. on Commun (Geneva, Switzerland)*. 1064–1070.
- Breddermann, T., & Vary, P. (2014). Rate-compatible insertion convolutional turbo codes: Analysis and application to LTE. *Wireless Communications, IEEE Transactions on*, 13(3), 1356-1366.
- Cristea B. (2010). Viterbi algorithm for iterative decoding of parallel concatenated convolutional codes. *Proceeding of 18th European Signal Processing Conference*, 1374-1378.
- Hanho Lee (2005). A high-speed, low-complexity reed-solomon decoder for optical communications. *IEEE Transactions on Circuits and Systems II*, 52 (8), 461-465.
- Höher P., Robertson P., Villebrun E. (1997). *Optimal and Sub-Optimal Maximum A Posteriori Algorithms Suitable for Turbo Decoding*. European Transactions on Telecommunications. 8 (2), 119–125.
- Ovechkin G., Satybaldina D., Tashatov N., Ovechkin P., Beisebekova A., (2014). Improving Performance of Non-Binary Multithreshold Decoder's Work Due to Concatenation. *Proceedings of the 18th International Conference on Communications (part of CSCC '14)*, 100–104.
- Ovechkin G., Zolotarev V., Satybaldina D., Ovechkin P., Tashatov N. (2014). The Performance of Concatenated Schemes Based on Non-binary Multithreshold Decoders. *Advances in Systems Science*. 240, 251-259.
- Richardson T., Shokrollahi M., and Urbanke R. (2001). Design of capacity-approaching irregular low-density parity-check codes. *IEEE Trans. Inform. Theory*.47, 638–656.
- Richardson, T., & Urbanke, R. (2008). *Modern coding theory*. Cambridge University Press.
- Robertson P., Villebrun E., Höher P. (1995). A Comparison of Optimal and Sub-Optimal MAP Decoding Algorithms Operating in the Log Domain. *Proceedings of the International Conference on Communications, (Seattle, United States)*, 1009–1013.
- Sharifi S., Tanc A.K., Duman T.M. (2015). Implementing the Han–Kobayashi Scheme Using Low Density Parity Check Codes Over Gaussian Interference Channels. *IEEE Transactions on Communications*, 63 (2), 337-350.
- Viterbi A.J. (1967). Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm . *IEEE Trans.*, IT-13, 260–269.
- Wilde M.M., Min-Hsiu Hsieh, Babar Z. (2014). Entanglement-Assisted Quantum Turbo Codes. *IEEE Transactions on Information Theory*, 60 (2), 1203-1222.
- Zolotaryov V.V. (1973). Device for decoding of the linear convolutional codes. Patent of USSR № 492878.
- Zolotaryov V.V. (2003). The Multithreshold Decoder Performance in Gaussian Channels. *Proc. 7th Intern. Symp.on Commun. Theory and Applications 7ISCTA'03 (St. Martin's College, Ambleside, UK, 13-18 July)*, 18–22.
- Zolotarev, V. V. (2006). The Theory and Algorithms of Multithreshold Decoding. *Under Scientific Edition of the Member Correspondent of the Russian Academy of Science, UB Zubarev, Moscow, Radio and Communications, Hot Line Telecom*.
- Zolotaryov V.V., Ovechkin G. V. (2010). Efficient Multithreshold Decoding of Nonbinary Codes. *Journal of Communications Technology and Electronics*, 55 (3), 302–306.