Improving performance of non-binary multithreshold decoder's work due to concatenation

Gennady Ovechkin, Pavel Ovechkin, Dina Satybaldina, Alina Beisebekova, and Nurlan Tashatov

Abstract—The abilities of non-binary multithreshold decoders (qMTD) for self-orthogonal codes in q-ary symmetric channel (qSC) with alphabet size q are analysed. The bit error rate performance of qMTD is compared with the decoder performance for Reed-Solomon codes and non-binary low-density codes. Simulation results shown that qMTD is considerably better than other methods of error correction regarding supported decoding error probability in case of comparable efficiency. A new concatenated coding scheme, which consist of two non-binary self-orthogonal codes (qSOC), is proposed. Comparisons with recent concatenated encoding scheme consisting of qSOC and non-binary Hamming are discussed, showing that the new scheme can offers better performance.

Keywords—concatenated codes, error-correcting codes, non-binary codes, non-binary multithreshold decoder.

I. Introduction

The increasing reliance on digital communication and the digital technologies as an essential tool in a technological society have placed error-correcting codes in a most prominent position. The effect from error-correcting codes applications can be expressed that they allow to work in case of significantly lowered level of a useful signal or in case of high error level. Error correcting methods can improve of many important features of communication systems (for example, increasing of communication range, decreasing of transmitter's power and antenna size). The real-world applications go far beyond the CD player to all of computing and data transmission technology, including hard disk drives, satellite communications, and digital television.

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At present time specialists take an active interest in non-binary codes operating with digital data at symbolic level, for example data with byte structure. Non binary (symbolic) codes are applied in channels with grouping errors and as a constituent element of concatenated codes. Symbolic codes are used for error correction protection information on various media (CD, DVD, Blu-ray and others).

Frequently, very elaborate precautions must be taken in present storage system to insure that they are free from errors. For example, magnetic tapes must be specially made and handled to guarantee the absence of defects, magnetic cores must be carefully tested to make sure that no defective cores get into an array. There are other storage methods whose development is hampered because of a common requirement for error free performance in all storage locations. With the use of error correction codes, such storage systems could be used, if they are sufficiently close to perfection, even though not perfect. Non-binary systems are be more erratic or noisy than binary storage systems, since each location must store one of many signals instead of one of two. This suggests that error correction codes may become essential with certain types of non-binary storage systems. The purpose of this paper is to develop codes for such storage systems and discover which decoding algorithms are most efficient.

Analysis of non-binary correcting codes and decoding algorithms showed that Reed-Solomod codes (RS) are very widely used in mass storage systems to correct the burst errors associated with media defects. Algebraic decoding algorithms allowing to correct up to half-distance errors are developed for RS codes [1], as well as more complex algorithms [2] providing correction of higher error number. But practically applied RS decoders can't ensure high decoding efficiency since it is impossible to build a decoder for long RS codes due to its highly difficult implementation. Lately many specialists have been developing non-binary low-density decoders (qLDPC) capable of very high efficiency [3]. However, the increased complexity of implementation, especially when alphabet size q is large, impedes their application in real systems.

In fact, J. Massey considered non-binary correcting codes and proved Theorems 1÷4 for these codes in [4]. But then he spoke negatively about these codes possibilities in sections 1.2, 6.2, 6.5, 6.6 and 8.2 of the same book and no longer

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engaged in the topic. A new iterative multithreshold decoding algorithm based on Massey algorithm [4] was developed in Russia [5-6]. A multithreshold decoder (MTD) is the development of the ordinary threshold decoder (TD) [4]. The each iteration of MTD differs from TD only presence "difference" register in which the information symbols changed by threshold element are marked. The value of this method shows that the majority algorithms provide almost optimal performance and have only linear computational complexity, as usually optimum methods are characterized by exponential complexity. Great interest to MTD is shown not only in Russia [7, 8].

Symbolic *q*-ary multithreshold decoders (*q*MTD) for *q*-ary self orthogonal codes (*q*SOC) is considered in [9-11]. The SER performance of *q*MTD is shown to be close to the results provided by optimum total search methods, which are not realizable for non-binary codes in general. *q*MTD decoders are compared with different decoders for non-binary RS and *q*LDPC codes. Research results presented in [11] show that *q*MTD for *q*SOC greatly exceed in efficiency decoders for RS codes and *q*LDPC codes being used in practice remaining as simple to be implemented as their prototypes – binary MTD**.** It is also very important not to use multiplication in non-binary fields during encoding and decoding as well as total independence of alphabet codes lengths from the size of applied symbols. Therefore, the *q*MTD for *q*SOC can be widely used in the processing, storage and transmission of large amounts of audio, video and other data.

This article reviews operation principles of symbolic multithreshold decoders, compares their efficiency with efficiency of other error correction methods and presents new approaches to improve *q*MTD efficiency. The other parts of the paper are arranged in the following way. Section II gives the concept of the q-ary multithreshold decoding. Section III shows the simulation results of *q*MTD efficiency comparison with efficiency of decoders for RS codes. A new concatenated coding scheme consisting of two non-binary self-orthogonal codes and its efficiency analysis are considered in Section IV. Section V shows the main conclusions of the paper.

II. Non-binary Multithreshold Decoding

Let's describe operating principles of *q*MTD for *q*SOC decoding. The description is given for *q*-ary symmetric channel (*q*SC) having alphabet size q, $q > 2$, and symbol error probability p_0.

Let's assume linear non-binary systematic convolutional or block self-orthogonal code with parity-check matrix **H** to be equal to binary case [6], i.e. it has only zeros and ones excluding the fact that instead of 1 there will be –1 in identity submatrix, i.e. **H** = [**P** : –**I**]. Here **P –** submatrix defined by generator polynomial for binary SOC; **I** – identity submatrix. Generator matrix of such code will be of **G** = [**I** : **P**T] type. This code can be used with any alphabet.size *q*.

The example of a scheme realizing the operation of encoding by block *q*SOC, given by generator polynomial $g(x)=1+x+x^4+x^6$, is shown in Fig. 1. Such code is characterized by the parameters: code length n=26 symbols, data part length k=13 symbols, code rate R=1/2, code distance d=5.

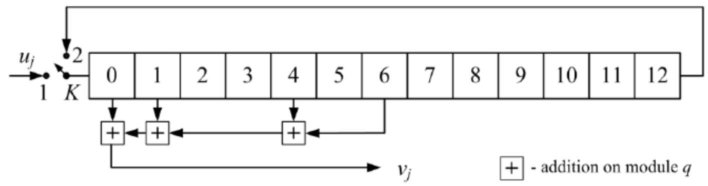

Fig. 1 encoder for block *q*SOC, given by polynomial $g(x)=1+x+x^4+x^6$

Note that for this *q*SOC during encoding and decoding operations only addition and subtraction on *q* module are necessary to be made. Calculations in non-binary fields are not applied in this case.

Let's assume that encoder has performed encoding of data vector ***U*** and received code vector ***A* = [*U*, *V*]**, where ***V* = *U* ·** **G**. Note that in this example and below when multiplication, addition, subtraction of vectors and matrices are made, module arithmetics is applied. When code vector ***A*** having the length *n* with *k* data symbols on *qSC* is transmitted decoder is entered with vector ***Q***, generally speaking, having differences from original code vector due to errors in the channel: ***Q*** = ***A*** + ***E***, where ***E*** – channel error vector of *q*SC type.

Operating algorithm of *q*MTD during vector ***Q*** decoding is the following [6].

1. Syndrome vector is calculated $\boldsymbol{S} = \mathbf{H} \cdot \boldsymbol{Q}^{T}$. Difference register ***D*** is reset. This register will contain data symbols changed by decoder. Note that the number of nonzero elements of ***D*** and ***S*** vectors will always determine the distance between message ***Q*** received from the channel and code word being the current solution of *q*MTD. The task of decoder is to find such code word which demands minimal number of nonzero elements of ***D*** and ***S*** vectors. This step totally corresponds to binary case.

2. For arbitrarily chosen decoded *q*-ary data symbol i_j of the received message let's count the number of two most frequent values of checks s_j of syndrome vector ***S*** from total number of all checks relating to symbol i_j, and symbol d_j of ***D*** vector, corresponding to i_j symbol. Let the values of these two checks be equal to h_0 and h_1, and their number be equal to m_0 and m_1 correspondingly when $m_0 \geq m_1$. This step is an analogue of sum reception procedure on a threshold element in binary MTD.

3. If $m_0 - m_1 > T$, where T – a value of a threshold (some integer number), then from i_j, d_j and all checks regarding i_j error estimation equal to h_0 is subtracted. This step is analogous to comparison of a sum with a threshold in binary decoder and change of decoded symbol and correction via feedback of all syndrome symbols being the checks for decoded symbol.

4. The choice of new i_m, $m \neq j$ is made, next step is clause 2.

Such attempts of decoding according to cl. 2…4 can be repeated for each symbol of received message several times [6]. Note that when implementing *q*MTD algorithm the same as in binary case it is convenient to change all data symbols consequently and to stop decoding procedure after fixed number of error correction attempts (iterations) or if during such iteration no symbol changed its value.

The example of *q*MTD implementation for encoder from Fig. 1 is given in Fig. 2.

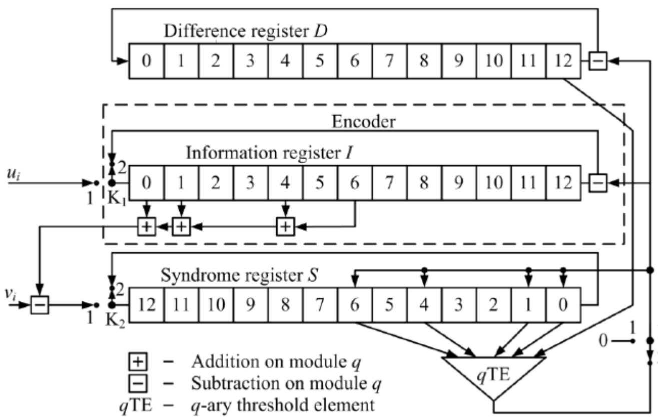

Fig. 2 *q*MTD for block *q*SOC

III. QMTD CHARACTERISTICS IN THE QSC

Let's compare characteristics of *q*MTD and other non-binary error correction methods in *q*SC. The volume of simulation in lower points of these graphs contained from $5{\cdot}10^{10}$ to $2{\cdot}10^{12}$ symbols which shows extreme method simplicity.

Dependencies of symbol error rate P_s after decoding from symbol error P_0 probability in *q*SC for codes with code rate *R*=1/2 are given in Fig. 3. Here curves *4* and *5* show characteristics of *q*MTD for *q*SOC with block length *n*=4000 and 32000 symbols when using 8-bit symbols (alphabet size *q*=256). As a comparison in this Figure curve *1* shows characteristics of algebraic decoder for (255, 128) RS code for *q*=256. As it follows from the Fig. 3, efficiency of *q*MTD for *q*SOC turns out to be far better than efficiency of RS code decoders using the symbols of similar size.

When code length in *q*MTD increases the difference in efficiency turns out to be even higher. Besides different methods to increase correcting capability of RS codes including all variations of Sudan algorithm ideally have the complexity of n_2 order. For the codes having the length of 32000 symbols this leads to the difference in complexity equal to 32000 times having at the same time little increase of error-correctness. This is shown in Fig. 3 by curve *3*, which gives Sudan algorithm features for (255, 128) RS code.

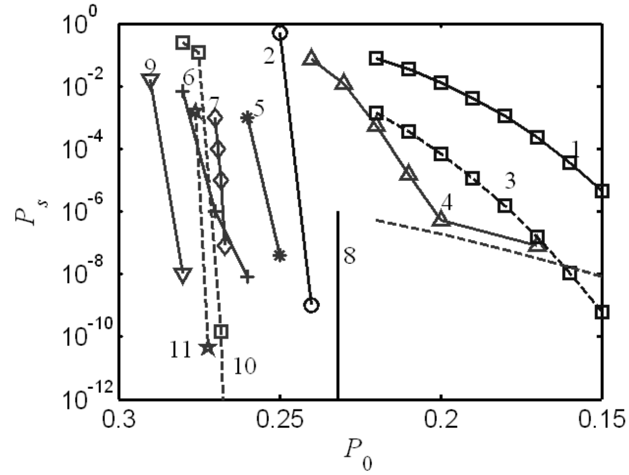

Fig. 3 characteristics of non-binary codes in *q*SC (*R*=1/2)

Note that even when using concatenated schemes of error correction based on RS codes it's not possible to increase decoding efficiency considerably. E.g., with the help of product-code having code rate 1/2, consisting of two RS codes with *q*=256 and several dozens of decoding iterations error rate less than 10^{-5} can be provided with error probability in the channel only equal to 0,18, which is considerably worse than when using *q*MTD.

Additional advantage of *q*MTD over other error correcting methods is the fact that it allows to work easily with symbols of any size providing high correcting capability. This is confirmed by curves which show characteristics of *q*MTD for code having the length equal to 32000 two-byte symbols (curve 6) and to 100000 four-byte symbols (curve 9). We should note that very simple to be implemented *q*MTD decoder for two-byte code with the length 32000 is capable to provide error-correctness not accessible even by RS code with the length of 65535 two-byte symbols (curve *2* in Fig. 3), the decoder for which is not to be implemented in close future. Besides, *q*MTD for four-byte symbols even surpasses in efficiency more complicated decoder of *q*LDPC codes with the length of 100000 four-byte symbols which has the example of characteristics presented in Fig. 3 by curve 8 [10].

To achieve these results codes for *q*MTD should be chosen very thoroughly and the main criterion while choosing should be the degree of resistance to the effect of error propagation. In the codes optimizing process it's possible to improve efficiency of qMTD [12]. Particularly, characteristics of the code with *q*=256 and code rate 1/2 found in [12] are given in Fig. 3 by curve 7. It is clear that this code provides effective work in conditions of bigger error probabilities in *q*SC, than the codes known before (curve *5*), having the same complexity of their decoding. Noted that part of corrected errors in *q*SC is increased up to 26.5% without complication of decoder due to choice of best code structure.

IV. PERFORMANCE OF CONCATENATED SCHEMES OF ERROR CORRECTION BASED ON QMTD BASE

One of the ways to improve *q*MTD characteristics is to use it in concatenated encoding schemes.

The first concatenated encoding scheme consisting of inner qSOC and outer non-binary Hamming codes is considered in [11].

At the same time known non-binary Hamming codes have such features as the necessity to use extended Galois fields in the process of decoding as well as dependence of code length from alphabet size. As a result the application of such codes in offered concatenated scheme especially when the alphabet is big becomes too complicated. That’s why it could be offered to build nonbinary Hamming codes [11] on the basis of known binary Hamming codes. The proposed decoding algorithm for modified non-binary Hamming codes in majority of cases (approximately in 71% cases for q=256 [11]) is able to correct even two errors. And if it is use offered extended non-binary Hamming codes having in addition one general check on module q then two errors are practically corrected in all cases (in 99% of cases for q=256 [11]). The example of performance of offered concatenated scheme containing qSOC with R=8/16, q=256, d=17 and given extended non-binary Hamming code with the length N_2=128 is shown in Fig. 3 by curve *10*. At the same time total decoding complexity due to addition of extended non-binary Hamming code increases not more than 35 % [11]).

The second concatenated encoding scheme consisting of inner qSOC and outer qSOC is proposed in this paper. The codes form a symbolic code-composition.

Let's consider the concatenated encoding scheme and a decoding method.

Let the inner non-binary code C_1 has a minimum code distance d_1, code length N_1 and an information part length K_1 and the outer non-binary code C_2 has a minimum code distance d_2, code length N_2 and an information part length K_2.

First each informational sequence are put in a matrix of size $K_1 \times K_2$. The matrix is encoded on columns by code C_2, then result is encoded on rows with using of code C_1. As a result each matrix row is a code word of the inner code while each column is a code word of the outer code. The symbolic code-composition has a code length N_1N_2, an information part length K_1K_2, a minimal code distance is d_1d_2, a code rate $R = K_1K_2/N_1N_2$. After completion of coding procedure data from matrix are read out by rows and transmitted over q-ary symmetric channel (qSC).

Structure of the code word for the proposed concatenated encoding scheme is presented in Fig.4, where symbols marked by ***I*** correspond to the information symbols of the inner code, and symbols marked by $\boldsymbol{C}^{(2)}$ - checking symbols of the outer code. Checking symbols of the outer code for information symbols ***I*** are denoted by $\boldsymbol{C}^{(1)}$. Checking symbols of the inner code for checking symbols of the outer code are indicated by $\boldsymbol{C}^{(*)}$.

At the decoding of the concatenated code first qMTD performs decoding of non-binary inner qSOC (decoding by all rows), and then using another qMTD performed decoding of non-binary outer qSOC (decoding by columns corresponding to information symbols of the concatenated code).

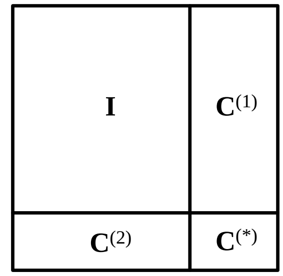

Fig. 4 code word of the concatenated code

For the proposed concatenated encoding scheme we form a rule according to which qMTD of the outer qSOK should work so that each change of decoded symbols a transition to a more plausible solution in comparison with previous decoder solutions takes place.

Let’s choose a non-zero value check h among all of the checking elements of syndrome and difference registers for the decoded symbols u_{ij} and find the value of the following expression

$$n_{ij}^{(h)} + \sum_{m\in\Omega_i} n_{mj}^{(s_{mj}^{(2)}-h)} \qquad (1)$$

If the sum (1) is a maximum and satisfies the (2),

$$n_{ij}^{(0)} + \sum_{m\in\Omega_i} n_{mj}^{(s_{mj}^{(2)})} < n_{ij}^{(h)} + \sum_{m\in\Omega_i} n_{mj}^{(s_{mj}^{(2)}-h)} \qquad (2)$$

then a distance between a received message and a code word of the concatenated code is reduced and consequently we pass to more plausible solution. Here $n_{ij}^{(x)}$ is a number of elements of the syndrome and differential registers for inner qSOC relevant to information symbols u_{ij}, which values are equal to x; Ω_i is a set of check numbers participating in decoding of outer code symbol i; $s_{mj}^{(2)}$ is an element of syndrome array of the outer qSOC decoder.

The characteristics of the proposed decoding method is presented in Fig. 3 by curve *11* which corresponds to the decoder performance of the concatenated code, consisting of the inner qSOK with d = 5 , R = 8/16 and the outer qSOK with d = 7 , R = 19/20 and q=256. Note that the proposed decoding method can be effective in conditions of bigger error probabilities in qCK (27.5% of bytes errors) that is unattainable for other practically realizable error correction methods of symbolic data.

V. Conclusions

In the paper the abilities of symbolic multithreshold decoders (qMTD) for self-orthogonal codes in q-ary symmetric channel (qSC) with alphabet size q are considered. The efficiency of qMTD algorithms in SER and in realization complexity is better than the efficiency of decoders for Reed-Solomon codes. This is defined the effective transfer of binary multithreshold decoding ideas on very simply organized non-binary codes of any.

A new concatenated coding scheme, which consist of two non-binary self-orthogonal codes (qSOC), is proposed. Comparisons with recent concatenated encoding scheme consisting of qSOC and non-binary Hamming are presented, showing that the new scheme can offers better performance.

Note that qMTD implementation complexity does not depend on alphabet size which enables to produce decoders of multithreshold type (including concatenated encoding scheme) efficiently correcting errors even in multibyte symbols (for example, in four byte symbols and more) for which development of other decoders is very difficult. Thus qMTD can replace Reed-Solomon codes in different data transmission and data storage systems.

Great deal of additional information on multithreshold decoders can be found on websites [18].

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