Improving Performance of Multithreshold Decoder for Self-Orthogonal Codes

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Abstract—**The article discusses self-orthogonal error-correcting codes (SOC) for the decoding of which multithreshold algorithms (MTD) are usually used. To decode SOC the algorithms used for low-density parity-check (LDPC) codes can also be applied. The article shows that using a min-sum decoder for SOC over a channel with additive white Gaussian noise (AWGN) in case of binary phase shift keying allows to receive additional coding gain about 1..1,5 dB in comparison with MTD usage. At the same time computing complexity in a min-sum algorithm turns out to be 6…7 times higher than MTD. For SOC decoding the work offers a combined decoder including the elements of MTD and min-sum algorithms. The first several decoding iterations require the usage of min-sum decoder while later MTD is added. The results of offered decoder simulation show about 1 dB increase of coding gain in comparison with MTD for SOC over a channel with AWGN with binary phase shift keying in case of twofold increase of computing complexity. The gain received depends on the SOC used, the number of min-sum decoding iterations and MTD.**

Keywords-self-orthogonal codes, multithreshold decoder, LDPC codes, min-sum decoder, coding gain, combined decoder

I. INTRODUCTION

Error correction in communication channels is usually made with the help of error-correcting codes. At present a coding theory offers large choice of codes and corresponding decoders [1], being characterized by different efficiency of error-correction. One of the challenging code classes can be self-orthogonal codes (SOC) [2], for the decoding of which multithreshold decoders (MTD) closely described in [3] are usually used. Such decoders provide close to optimal decoding of correctly chosen codes with linear complexity (when code length is considered), depending on the code distance of the codes used and the number of decoding iterations [4..7].

Besides, SOC high efficiency of error-correction can be provided by low-density parity-check (LDPC) codes [8]. These codes represent linear codes determined by parity-check matrix containing basically zeros and several number of ones. LDPC codes apply quite efficient decoding algorithms operating with Tanner graph of code [9, 10, 11] and allowing to provide error-correctness at the level of noise being a few tenths of dB lower than possible. One of the simplest if taken from computing viewpoint is a min-sum algorithm [12].

Self-orthogonal codes analysis shows that they have low-density parity-check matrixes. So it is possible to use LDPC codes decoders (e.g. min-sum decoder) for decoding of SOC [13]. Considering this, in comparison with existing LDPC codes SOC will be less complicated in coding.

The work [13] shows that using a min-sum algorithm for SOC decoding it is possible to receive additional coding gain equal approximately to 1 dB in comparison with MTD having some increase of complexity which is not always acceptable. The work given offers SOC decoding algorithm allowing to increase coding gain in comparison with MTD, the complexity being lower than min-sum algorithm.

II. MULTITHRESHOLD DECODERS OF SELF-ORTHOGONAL CODES

Self-orthogonal codes (SOC) are the subclass of codes allowing majority decoding [2]. SOC are characterized by the fact that the system of all checks controlling the errors in any information symbol is itself an orthogonal one concerning this error. It should be noticed that the orthogonality of a check system concerning an error is understood as the participation of this error in all system checks so that no other error participates in more than one check. Usually SOC are made with the help of generator polynomials $g(x)$, differential triangles (a set of differences between all polynomial degrees with nonzero coefficients) of which don't contain equal elements. Simplest block SOC are characterized by code distance d, equal to the number of nonzero components of generator polynomial increased by 1. The number of information symbols k in code block with code rate R=1/2 is equal at minimum to $2m+1$, where m – maximum degree of generator polynomial. The length n of such code equals $2k$.

To implement coding of SOC simple circuits built on the basis of shift registers can be used. The example of coder for a block SOC is represented in Fig. 1 [3]. A given code is characterized by n=26, k=13, R=1/2, d=5, $g(x)=1+x+x^4+x^6$ parameters. For SOC decoding multithreshold algorithm [3], being the development of usual Massey threshold decoder [2], can also be used. The example of MTD decoder for the block SOC specified by polynomial $g(x)=1+x+x^4+x^6$ is presented in Fig. 1 [3].

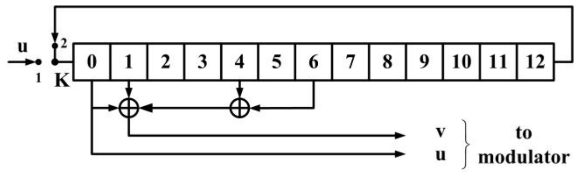

Fig. 1. Coder for block SOC with R=1/2, d=5 and n=26

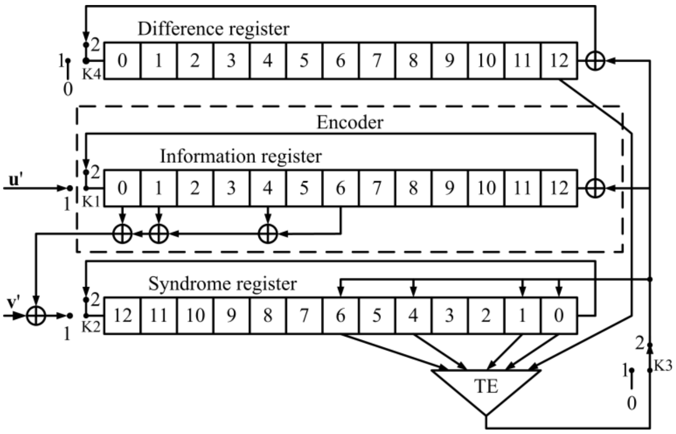

Fig. 2. Multithreshold decoder for the block SOC with R=1/2, d=5 and n=26

Up to now a number of approaches allowing to increase MTD efficiency has been elaborated [3, 15, 16].

III. MIN-SUM AND COMBINED DECODING FOR SELF-ORTHOGONAL CODES

The results of the efficiency research for multithreshold decoding of self-orthogonal codes presented in [3-7] show that MTD actually provide close to optimal decoding of correctly chosen codes with linear from code length implementation complexity. For MTD implementation complexity is proportional to coding distance d of codes applied and the number of decoding iterations I. The number of arithmetic operations required for decoding of one data bit is approximately equal to [13]:

$$N_{mtd} \approx I(d+2)+d-1 \approx (I+1)(d+2)\ .$$

Besides SOC the high coding gain can be provided with LDPC codes, offered by R. Gallager [8]. For these codes efficient iterative decoding algorithms operating with Tanner graph of code [9] are known. Initially belief propagation (BP) algorithm was used for decoding of LDPC codes. The efficiency of this algorithm is close to optimal algorithm, although BP algorithm has great implementation complexity. Due to this, a amount of works devoted to the analysis of LDPC codes decoding quality as well as implementation of modifications in BP algorithm with the aim to decrease its complexity has appeared [10, 11, 12]. When seen from the viewpoint of computation, one of the simplest ones is a min-sum algorithm [12].

The analysis of SOCs shows that they also have low-density parity-check matrix [13]. E.g., check matrix for block SOC specified by the polynomial $g(x)$=1+x+x^4+x^6 is as follows

$$\mathbf{H}=\begin{bmatrix}11001010000001000000000000\\01100101000000100000000000\\00110010100000010000000000\\00011001010000001000000000\\00001100101000000100000000\\00000110010100000010000000\\00000011001010000001000000\\10000001100100000000100000\\01000000110010000000010000\\10100000011000000000001000\\01010000001100000000000100\\00101000000110000000000010\\10010100000010000000000001\end{bmatrix}=[\mathbf{P}:\mathbf{I}].$$

For similar codes the number of ones in the lines and rows of submatrix P equals d–1 (where d is code distance), and the number of ones in the lines and rows of submatrix I equals 1. Tanner graph for the SOC with generating matrix **H** is represented in Fig. 3. Therefore, for SOC decoding it is possible to apply iterative decoders for LDPC codes, in particular, min-sum decoder [13]. In comparison with existing LDPC codes SOC will have much less coding complexity which is important for many digital radio communication systems. Besides, given decoding techniques can be applied for convolutional SOC which are possible to be used for continuous data flow transmission.

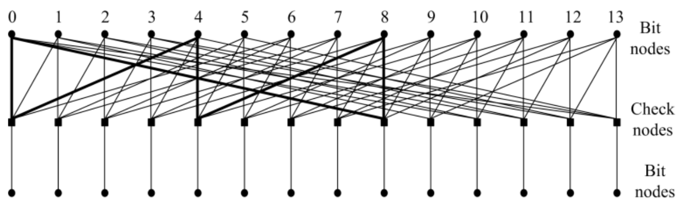

Fig. 3. Tanner graph for the SOC

Implementation complexity per iteration of a min-sum decoder for SOC with code rate 1/2, code length n, information length k and code distance d is defined in the following way:

– the first step is to calculate message values from n bit nodes to the check one connected with them. On using optimization represented in [11], this stage requires $N_1 = 2(d-1)k$ additive equivalent operations;

– the next step is to calculate message values from each k of check nodes to bit one connected to them. This stage requires $N_2 = (6d-4)k$ additive equivalent operations.

After the last iteration leads to the formation of decoder decision. This stage requires $N_3 = (d-1)k$ additive equivalent operations.

As a result of the whole coding block decoding with I decoding iterations the number of performed operations equals $N_{ms}^{k} = I(N_1 + N_2) + N_3 = I(8d-6)k + (d-1)k$.

Then decoding one information bit by a min-sum algorithm requires $N_{ms} = I(8d-6)+(d-1)$ additive equivalent operations.

The comparison of implementation complexity of a min-sum decoder and MTD while performing equal number of decoding iterations for typical values of code distance has shown that a min-sum decoder performs approximately 6…7 times larger number of operations during decoding than MTD. This means that decoding complexity during the application of 6 iterations of MTD algorithm is comparable to the complexity of only one iteration of a min-sum algorithm. At the same having equal number of iterations a min-sum algorithm provides a larger energy gain (about 1 dB) than MTD [13].

To increase the efficiency of SOC decoding but to leave low complexity the combination of decoding algorithms under discussion (min-sum and MTD) can be applied. It is recommended to use a min-sum algorithm in the first decoding iterations as it allows to work with larger noise. After several iterations of a min-sum algorithm MTD can be used. The circuit starts efficient decoding of the code applied in the conditions of higher noise level than when used with MTD but its complexity increases twofold in comparison with MTD. The structure of the offered decoder is represented in Fig. 4.

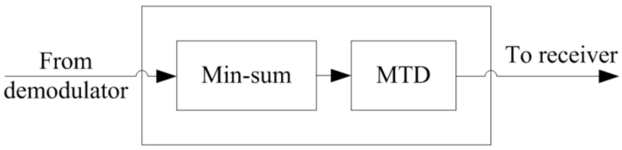

Fig. 4. Structure of combined decoder

Undoubtedly, instead of a min-sum algorithm the combined decoder can use other decoding algorithms developed for LDPC codes. It should be noted that the efficiency and complexity of the proposed decoder are influenced by SOC applied, the number of a min-sum algorithm iterations and the number of MTD iterations. In what follows, we give the research of an offered decoder.

IV. SIMULATION RESULTS AND THEIR DISCUSSION

First, let us consider the characteristics of different decoding algorithms for a block SOC having code rate R=1/2, code distance d=9, code length n=20748 bits. These characteristics are received for the AWGN channel with using binary phase shift keying. Fig. 5 shows the curves «MTD(30)» and «min-sum(30)» which reflect MTD and min-sum algorithms using 30 decoding iterations characteristics for this SOC. Curves «min-sum(3)+MTD(30)», «min-sum(5)+MTD(30)», «min-sum(7)+MTD(30)» and «min-sum(9)+MTD(30)» represent the characteristics of combined decoder when first 3, 5, 7 and 9 min-sum algorithm iterations and later 30 MTD iterations are used correspondingly. As a comparison, Fig.5 gives the characteristics of min-sum decoding with 8, 10, 12 and 14 iterations (curves «min-sum(8)», «min-sum(10)», «min-sum(12)», «min-sum(14)»), implementation complexity of which corresponds to the complexity of combined decoders under review. The charts show that the more iterations with a min-sum algorithm are used in the component decoder, the larger is the gain in comparison with MTD, and the larger is implementation complexity. If more than 9 min-sum iterations are used, the combined decoder has no gain when compared only with a min-sum algorithm having equal implementation complexity.

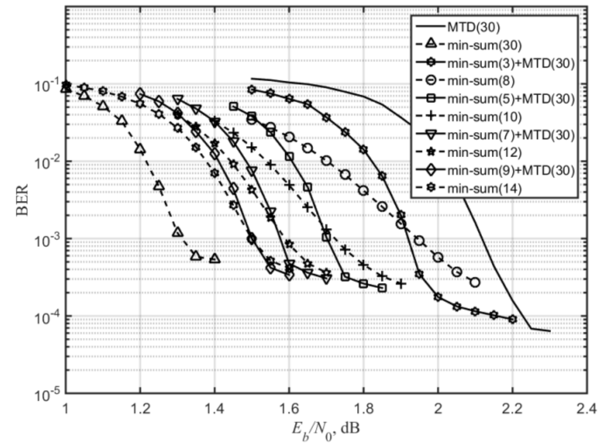

Fig. 5. Characteristics for SOC with n=20748 and d=9

Consequently, for this SOC using a combined decoder with 9 min-sum iteration and 30 MTD iterations we received the gain about 0,75 dB in comparison with usual MTD. Proposed decoder turns out to be a little more twice as hard as MTD in the number of operations performed. In comparison with the usage of a min-sum algorithm only we have 0,3…0,35 dB decrease of efficiency at twofold complexity decrease.

The results based on the study of SOC min-sum decoding efficiency in [13] show that a min-sum decoder for different SOC works similar to MTD. Eventually min-sum decoder application can allow usage of efficiency increase approaches developed for MTD such as the application of parallel coding for codes with allocated branches, codes resistant to error propagation, concatenation with simplest outer codes, etc. [3]. In other words, good SOC for MTD are simultaneously considered to be good SOC for a min-sum decoder. To increase the efficiency of proposed decoder we use block SOC with allocated branches having the length n=31824, code rate R=8/16 (a code has 8 information and 8 check branches) and code distance d=17, the structure of which is represented in the table in Fig. 6.

Table cells with the size 8x8 give the number of j-th information branch symbols taking part in the formation of i-th check code branch. The sum of all numbers in i-th line determines dimension of SOC checks built on i-th check branch taking into consideration information symbols of all eight information branches. These sums are given in the right table column. The sum of numbers in j-th column is the total number of checks relative to the symbols of j-th information branch which determines code distance for this branch. For the code considered, the checks of branch 8 having extremely big dimensions at big noise with the probability close to 0,5 are not correct. Therefore in the first decoding iterations as well as in the course of MTD and min-sum algorithm application the checks of this branch are not used. Besides efficiency increase this additionally reduces the complexity of decoding.

2	2	1	1	1	0	0	0		7
2	2	1	0	1	1	0	0		7
1	2	2	0	1	1	1	0		8
1	2	2	0	1	1	1	1		9
2	1	2	1	0	1	1	1		9
2	1	2	1	0	0	1	1		8
2	2	2	1	0	0	0	1		8
4	4	4	12	12	12	12	12		72

Fig. 6. The structure of SOC with allocated branches

Let us further consider the characteristics of combined decoder for SOC considered with total number of iterations equal to 35. In Fig. 7 curves «MTD(35)» and «min-sum(35)» represent decoder characteristics when using only MTD or min-sum decoding algorithms with 35 iterations. The names of other curves in Fig. 7 include the names of decoding algorithms applied and the number of iterations. The charts given show that having the same number of iterations (35 iterations) MTD efficiency is 1,4 dB less than min-sum algorithm efficiency, but implementation complexity of MTD is 7 times less. When using combined decoder we get the gain in the range between 0,2 and 1,1 dB in comparison with MTD with two-threefold increased decoder complexity. Increasing the number of min-sum algorithm iterations leads to the increase of the efficiency of the decoding circuit offered.

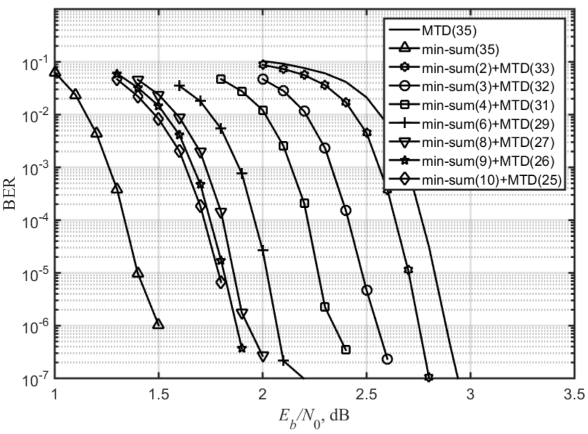

Fig. 7. Characteristics for SOC with n=31824 и d=17

Thus, for this SOC using the combination of MTD with 26 iterations and min-sum algorithm with 9 iterations we have the gain up to 1,1 dB in E_b/N_0 compared to MTD and we have loss the efficiency on 0,3…0,4 dB compared to min-sum decoder at threefold reduction of complexity.

V. Conclusion

The work offers a combined decoder for self-orthogonal codes. The results given have shown that when adding several min-sum decoding iterations into MTD decoding circuit the efficient decoder operation approaches channel bandwidth at only a few tenths of dB only slightly increasing its implementation complexity. The efficiency of the combined decoder depends on the SOC used, the number of min-sum iterations and MTD algorithms applied in it.

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