Usage of Divergence within Concatenated Multithreshold Decoding Convolutional Codes

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***Abstract*—The present paper has considered multithreshold decoders for self-orthogonal codes providing a near-optimal efficiency of the error correction under linear computational complexity. New divergence principle used within construction and decoding convolutional codes has been discussed. The paper has shown that usage of such principle allows significantly approximating an area of the decoder effective operation to the channel capacity. Application of obtained self-orthogonal codes within construction of concatenated convolutional codes where parity-check codes are used as the outer codes has been indicated, as well as simulation results for the obtained concatenated construction have been represented.**

Keywords—communication system, error-correction coding, self-orthogonal code, concatenated code, parity-check code, multithreshold decoder, divergent encoding, divergent decoding, decoder complexity, coding gain

I. Introduction

One of main issues occurred within development of system for transfer and storage of information is an issue to ensure error-free information transfer over channels with noise. Error-correcting coding is usually used for solution of the present issue. In recent decades development of such methods has reached significant successes. At present there is a range of error-correcting codes and their decoders, for example, low-density parity-check (LDPC) codes [1], turbo codes [2], polar codes [3] providing operation close to the channel capacity. One of the most effective algorithms from the point of view of efficiency and implementation complexity are multithreshold decoders (MTD) for self-orthogonal codes (SOC) being a development of the threshold decoder and allowing decoding both block and convolutional SOC [4..10] with linear complexity. Base of MTD operation is iterative decoding within which process it becomes possible to approximate to the solution of optimum decoding in the enough wide range of code rates and signal to noise ratios. Besides, MTD save simplicity of a usual threshold decoder. SOC decoded with MTD is a sub-class of codes permitting majority decoding. Simple schemes based on shift registers can be used for implementation of SOC encoding and decoding [4].

Main achievements of the Optimization Theory (OT) of error-correcting coding represented in [4, 9] in the framework of which MTD are developed, confirm that MTD constructed according to new postulates of this theory have currently reached a rather high level of efficiency with moderate complexity. Contemporary possibilities of MTD are such that these algorithms provide an bit error rate (BER) less than 10^{-5} under the signal to noise ratio $E_b/N_0 \approx 1.3$ dB in Gaussian channels at usage of the Binary Phase Shift Keying (BPSK) and 16-level integer decisions of the demodulator. It is enough difficult to arrange such efficient operation, for example, decoders for LDPC codes under the same conditions. On the other hand, a possibility to implement MTD on the basis of technical solutions [11] completely removes an issue of the decoder throughput because it allows remaining high decoding performance at any channel transmission rates, including ones that are multiply higher than 1 Gbit/s [4, 6, 9]. In addition, resources for improvement MTD performance have not fully exhausted that allows expecting from them further improvement of operation efficiency under high levels of noise in future. The present paper has suggested a new direction for the MTD development which can help to approximate an area of the MTD efficient operation to the channel capacity.

The rest part of the paper is arranged as follows. Section 2 describes a principle of divergence used under construction and decoding convolutional codes. The performance of new codes and methods for decoding are considered in Section 3. Section 4 is devoted to usage of the divergence principle within decoding concatenated codes and discussion of their performance. Section V concludes this paper.

II. Divergence Principle within Decoding Convolutional Codes

Divergence principle within design of codes and algorithms for their decoding [12] has been created within the process of development the Optimization Theory (OT) of error-correcting coding [4, 9] allowing implementing a new level of the effective errors correction based on MTD and other algorithms. Divergent methods for code construction and developing algorithms for their decoding create a new style of the gradual non-concatenated extension of the code distance in convolutional codes using the simplest iterative decoders, for example, MTD. This approach becomes especially important for a low signal to noise ratio. Some complexity of the initial method implementation always remains and higher resulting efficiency of coding is implemented, if, for example, codes permitting multithreshold decoding are used [4]. In this case, such efficient decoding becomes actually possible which even at high channel noise coincides by the error probability with optimum decoding for used long codes which is usually implemented on the basis of complete enumeration, for example, as in the case of Viterbi algorithm [12]. However, MTD complexity which even under divergent encoding become the simplest schemes with majority decoding, remains theoretically minimally possible, linear from the code length. It allows using long codes that under optimum decoding provided with MTD leads to the best performance at a very high noise level today.

Then we indicate main principles of divergent encoding and decoding. Let's consider a circuit of the simple encoder for convolutional code of rate R=1/2 represented in Fig. 1. Figuratively speaking, it consists of a shift register in the left part of which cells are grouped from outputs of which content values arrive to inputs of the modulo 2 adder from output of which check symbols of the code pass to the channel. To simplify the description, we assume the code to be systematic. That is why one informational symbol from a zero cell of the shift register leaves together with a current check symbol of the code into the channel. Fundamental moment for description of the present encoder operation is a presence of one more cell far in the encoding register right part which content also arrive to the modulo 2 adder input from which data move to the channel. Of course, a code can be non-systematic and a number of cells in the register right part from which data are sent to the multi-input modulo 2 adder can in the general case be enough high. But now let's confine to analyzing represented circuits.

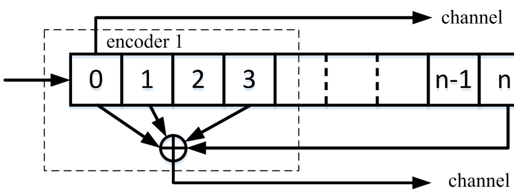

Fig. 1. Encoder for the divergent code

Fig. 2 shows a decoder for the convolutional code corresponding to the encoder shown in Fig. 1. It was constructed according to the MTD ideas and contains two threshold elements located in the left (TE1) and right (TE2) parts of the decoder. TE1 and corresponding parts of informational and syndrome registers with which it interacts, are marked by a dotted rectangle and called as decoder 1 (D1).

Full decoder with TE2 in the right part of decoder registers is similar to D1. But also additional check arrives to the input TE2 which appears in the decoder much later than symbols of the compact group of checks connected with TE1.

Under operation in the channel TE1 makes decisions on informational errors based on only its own group of checks. If channel noise and code are selected correctly then after TE1 density of such errors will be less than before such threshold and reaching TE2 these errors will be corrected in most cases according to principles of MTD operation. Since a number of checks arriving to TE2 inputs is one more than in TE1 then correcting possibilities of TE2 will be higher that allows intensifying the correction process because TE2 operates with the code where a minimal distance d was increased in one unit in comparison with TE1. It is important that it has been achieved without methods of concatenation which takes redundancy from the first code (and TE1) that notably reduces correcting possibilities of the first decoder.

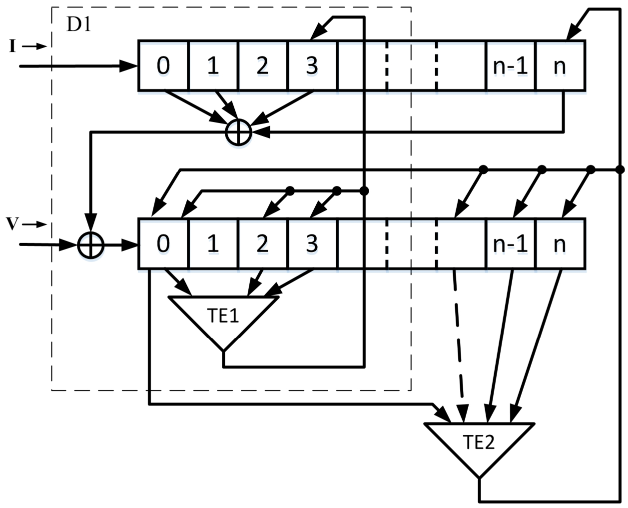

Fig. 2. Decoder for the divergent code

It is obvious that the suggested code can be the first part of the even longer code with a similar structure itself. Then on two such conditional "concatenations" of encoding/decoding minimal distance d has been already increased in 2 etc. In reality, such schemes successfully operate, they correspond to principles of the MTD operation and main theorem of the multithreshold decoding [4] showing satisfactory results.

Actually, the obtained scheme of decoding became much more complicated since effect of the code distance (extremely valuable resource) growth cannot be obtained so easily. The first decoder in Fig. 2 allows a part of errors which it has not corrected to pass to the right to TE2. And then these errors enter into the syndrome register from cell n through modulo 2 adder. It means that TE1 operates at a slightly increased level of noise that worsens its characteristics. But if TE1 copes with this increased flow of errors and slightly worsens its own characteristics and TE2 helps TE1 then we can expect that they mutually cope with such more complicated flow of errors that allows continuing an analysis of this scheme for determination of its capabilities at the high level of noise.

Significant peculiarity of the divergent decoding style is a gradual structural complication of the decoder, for example, of majority type, as well as multiple application of this approach. In this case within construction of the code and its decoder, the already constructed structure of the decoder is again considered as some initial decoder which is included itself to the external scheme for divergent decoding of the higher level in future. Efficient application of the considered divergent principle of the decoding procedure arrangement is this multiple (three-, five- and more-multiple) gradual expansion of the code distance which is further fully implemented, for example, by MTD. Under correct design of the decoding procedure according to divergent principles MTD within execution of the error correction process increases reliability of decisions to the level of optimum decoding for used codes as under usage of usual codes.

Considered approach to arrangement of the decoding procedure obviously leads to increase of duration and complexity of the design process, research and adjustment of created decoders of such type, notable growth of the used code length, as well as increase of decision-making delays since a number of required decoding iterations, including under usage of MTD, also notably grows. However, as it has been repeatedly noted in recent literatures devoted to error-correction coding [4, 14, 15], within increase of the channel noise level all real decoding methods require some complication of the methods themselves, procedures of their design and, most importantly, significant expansion of the used code length both in block and convolutional variants of implementation.

Regarding the divergent style of decoding, its mostly important property is that after completion of design and adjustment of the algorithm using this enough efficient approach, actually in most cases the developed method for decoding remains as simple as the original algorithm taken as a base. In particular, if MTD is accepted as a base of the design decoder of divergent type then the resulting scheme of the type being traditional for convolutional algorithms with a lot of threshold elements practically always remains the same as MTD with rather clear principles of operation that is also important for training of specialists who will control and assists operation of such coding systems in future. Actually the only distinction between the divergent MTD and its standard classical form is usually that threshold elements used at various decoding iterations use different sets of checks executing the same simple operation for change of syndrome symbols and decodable symbols on the basis of their values for decision making. Number of such various sets can reach 3...5 variants out of the total number of about 20 checks that does not complicate the algorithm. Moreover, since at the first iterations threshold element as it follows from the description of divergence principles, use a small part from the whole set of checks, the MTD developed according to such new principles performs even a less number of checks summations on the general threshold element collection than if all threshold elements use all checks. So, divergent MTD remain really rather simple decoding systems, but as it was assumed under their development, understanding of the divergent decoding essence and proper design of corresponding decoders have allowed creating algorithms of the threshold type in reality which successfully operate close to the Shannon bound, i.e. almost at maximum possible noise level.

III. SIMULATION RESULTS

Let's consider the most effective new results of the divergence principle application obtained for Gaussian channels. Fig. 3 shows dependencies of bit error rate for various decoding algorithms as a function of the bit signal to noise ratio E_b/N_0 in the Gaussian channel with using BPSK modulation and a demodulator forming 4-bit soft decisions. Vertical C=1/2 indicates the signal to noise ratio at which the channel capacity C is equal to the code rate R=1/2 of all used codes. Curve *1* reflects possibilities of the widely used Viterbi algorithm for convolutional code with the encoder register length K=7. Curve *2* corresponds to a concatenated code based on the convolutional code with K=7 and Reed-Solomon code. Curve *3* is mentioned for the min-sum decoder for LDPC code of DVB-S2 standard with length 64800 bits implemented in the Comtech modem CDM-710. Curve *4* corresponds to MTD successfully operating under code rate R=1/2 in the binary Gaussian channel at E_b/N_0=1.2 dB, i.e. at a distance of only 1 dB from the Shannon bound. Let's note that it is a result of implementation of the three-level divergent scheme for self-orthogonal systematic code with minimum distance d=21. Decoder operation requires not more than I=160 iterations. Value of the decoding delay at convolutional coding is less than 6 Mbit. In the case if majority-decoded code with the same code length was constructed not taking into account ideas of the divergent decoding, then even using methods of parallel concatenations [4] and other code constructions it would be possible to ensure MTD operability in the Gaussian channel at noise level more than 2.9 dB. Thus it can be argued that OT and MTD have become a simple and technological solution of the Shannon issue – effective and simple decoding at the maximum allowable noise level.

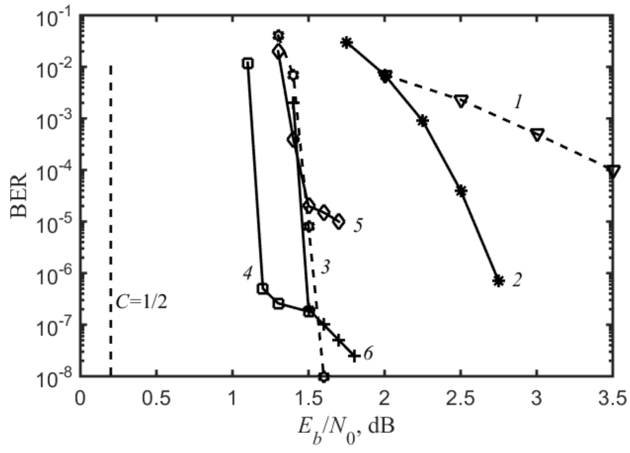

Fig. 3. Performance for error-correction methods over Gaussian channel

It is worth noting that performance of MTD decoders for symbol codes represented in [4, 9, 16] successfully operating at probability of independent errors in the channel $p_0 \lesssim 0.3$ and also the decoder for erasures recovering use the same nested divergence structure as in the binary case that allows them providing unique performance on the basis of the simplest MTD.

IV. CONCATENATED ENCODING SCHEMES AND SIMULATION RESULTS

After consideration of importance and efficiency of non-concatenated divergent code structures, let's analyze possibilities of traditional sequential concatenated schemes. As within creation of long-used concatenated codes, such methods can be successfully applied in close to the Shannon bound for any code rates and channel models with independent errors. However, there is an important restriction on codes forming part of the concatenated scheme for such systems which should be used close to the channel capacity C. It is that an outer code should have a minimum redundancy, i.e. code rate $R_2 \lesssim 1$ since otherwise an inner code with rate R_1 will be unable to operate only because it may turn out that $C<R_1$. So, it becomes obvious that the most acceptable outer codes for the concatenated sequential scheme are parity check codes (PCC) having a minimum redundancy but doubling the code distance of the concatenated code since they have this distance $d_2=2$. In this case, the coding gain can be increased up to 3 dB in comparison with the case if only the inner code is used. Such approach is more important because PCC really interact with a lot of types of inner codes very simply.

However, the most important point within concatenated codes development is that their length N is a product of lengths of two constituent codes: $N=n_1n_2$. And as it was mentioned before, redundancy of the second outer code should be small then it is obviously necessary to fulfill the condition $n_2>>1$. But then length N of the whole concatenated code should grow in hundred times in comparison with the original code. If we choose smaller values n_2, characteristics of coding will obligatory move away from the Shannon bound simple because of losses due to increase of the code rate R_2.

Simple circuit of concatenation being easy for implementation is suggested below. For this purpose let's firstly consider a classical two-dimensional scheme of concatenation based on PCC and codes with majority decoding [4]. If decoding of the inner code is implemented on the basis of MTD and considered that MTD decision practically coincides with the optimal one and its bit error probability is $P_b(e)$, then bit error probability of the whole concatenated scheme will be quite accurately estimated by expression $P_{KK} \approx 2n_2P_b^2(e)$. The most significant fact for construction of the required concatenated scheme is that two-dimensionality of concatenated schemes was obligatory because of algebraic codes within incorrect decoding of which a rate of errors inside the code block was significant. But errors under usage of MTD more often occur singly that is determined by a structure of used codes. Then we can apply a one-dimensional scheme of encoding by a low-redundant code and then the same informational flow with rare insertions of parity check bits is encoded with the inner code. Since it turns out that MTD errors are not only single but almost independent then we can expect that the above represented estimation P_{KK} will be rather accurate for a new concatenated code of length $N_{KK2} \approx (n_1+n_1/n_2)$.

However, if inner codes for MTD decoding a concatenated code are not too long then it is possible that inside PCC block we found a lot of such pairs of informational bits that they are simultaneously summands while calculation some check bits. For inner SOC with code distance d_1 it leads to the fact that payment for the "short" concatenated code is that distance of the concatenated code being equal to $d_{kk}=2d_1-2$ but not to $2d_1$. This circumstance sometimes reduces a decoder error probability of the concatenated code up to 5 times. But since an absolute value of the decoder error probability, as it was shown above, is rather small then such payment for decrease of the concatenated code length in 2 orders is rather applicable.

Taking into account above mentioned peculiarities of concatenation, the concatenated scheme has been developed showing its high efficiency close to the Shannon bound. SOC with $R_1=1/2$ and $d_1=15$ was chosen as an inner code which was decoded sub-optimally at $E_b/N_0 \geq 1.5$ dB as curve *5* in Fig. 3 shows. Number of decoding iterations is 80 and total decision delay is less than one million informational bits that is a very good result for such heavy operational conditions and according to these parameters. Volume of statistics for three right points on the diagram exceeded 5×10^7 bits and all errors were uncorrectable under optimal decoding.

Concatenated code using PCC with $(n_2, k_2, d_2) = (192, 191, 2)$ has been constructed according to above described rules of the "short" code formation and it had a code distance $d_{kk}=28$. According to [4], MTD firstly operates in the concatenated code and reduces an error probability up to the optimum level as in the first code. Five decoding iterations were added to the concatenated code. Decoding delay was less than 1 Mbit. Latest 20 decoding iterations were arranged taking into account presence of PCC. At the same time, after completion of receipt of regular 192 bits of the outer code after successful parity check the decoder passed to following symbols decoding and if the check failed, the least reliable bit of the outer code was changed if it was single. The efficiency of the concatenated code is represented by curve *6* in Fig. 3. Volume of statistics at $P_b(e)<10^{-6}$ is more than 2×10^8 bits and all errors of the decoder corresponded to errors of optimal decoding. As it should be from description of the present scheme, the absolute majority of errors in concatenated code was double locating at a distance less than n_2, and a small rate of errors turned out to be single. Presence of only two types of errors testifies that even concatenated code is decoded with MTD practically optimally. Losses of energy due to presence of the outer PCC are about 0.02 dB. It allows stating about correct arrangement of the effective decoding close to the Shannon bound.

In conclusion, let's note that success of the concatenation task solution at a low E_b/N_0 on the basis of rather difficult example allows evaluating a possibility of concatenation based on MTD shown in Fig. 3 by curve *4* which effectively operated at $E_b/N_0=1.2$ dB that is an excellent result. Our estimate shows this concatenated MTD would provide BER significantly lower than 10^{-9}. More accurate estimation for this code can be obtained using software [17] or hardware [9] prototype which is easily created based on experience of the successful design of MTD using FPGA Altera. It is possible that real experiment would provide BER about 10^{-11}.

Finally, let's mention that suggested methods for concatenation of symbol codes and codes for erasure recovering, as in the case of binary codes correcting errors, are easily implemented on the basis of MTD and they increase reliability of decoding by the simplest means in many decimal orders.

V. CONCLUSION

The present paper has represented the suggested divergence principle used under construction and decoding convolutional codes in the framework of the Optimization Theory and also application of constructed codes in concatenated code constructions has been discussed. The paper has shown that codes and decoders developed on the basis of this principle are capable to ensure error probability about 10^{-6} and significantly less at signal-to-noise ratio 1.2 dB over the Gaussian channel with using the Binary Phase Shift Keying. Moreover, computational complexity of the decoder becomes in ten times less than complexity of other methods for error correction which can provide similar BER performance.

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