

Conclusion

Our scientific school on Optimization Theory have successfully carried out plans and has realized all his intentions to the fiftieth anniversary of the publication of amazing in its depth and at the same time for clarity of presentation of J. Massey's the book "Threshold decoding" [25]. Published for this anniversary, our monographs [4,5] have shown wide opportunities of the majority methods at the OT basis for all main types of classic channels studying in coding theory.

Very special anniversary of the 2018 linked in information theory with truly great paper by C. Shannon [14], which was published 70 years ago and gave rise to large-scale projects worldwide scientific and technical community to create those very necessary for our digital information world methods of errors corrections, the existence of which we had precisely and accessibly told in this work. This monograph is devoted to successful completion of searching such methods which, as suggested by the supporters of our research areas, finally completely solved great problem stated in that anniversary publication.

Our book gives a complete solution of C. Shannon's task of extremely simple and efficient error correction in digital arrays for all major types of communication channels. This problem was successfully solved up to the realms of noise that are directly close to the bound, known as a channel capacity, which this great American scientist has clearly defined. This bound is unachievable as the speed of light for material bodies, since it's absolutely elasticity. At the present time for most of main channels in coding theory technologies of decoders creation already exist successfully working directly in the nearest energetic domain to this bound. It means that a fundamental and very complex process of finding a solution to the problem of Shannon is completed. That quite a small remaining distance to his bound, future researches can pass (of course only partly!) using methods that are already developed or yet to be created in the process of further research in Optimization Theory (OT) of error-correcting coding. The infinitely high elasticity of capacity will continue to be very reluctant to allow the operation of the decoding algorithms at extremely high noise levels. But the results obtained at the energy/noise ratio and probabilistic characteristics of decoders for roughly a hundred code clusters (typical sets of code and channel parameters) are now quite sufficient to consider the problem completely solved and to create thousands of other similar high performance systems with a very reasonable complexity of implementation. Of course, for many other specific code clusters they will need to create by methods OT already known or future, new, custom, coding and subsequent decoding configurations. Widely presented set of design tools for research and configure codecs (coders and decoders) almost always allow them to create the required coding system acceptable at the complexity, efficiency and reliability to work close to a channel capacity .

Algebraic coding theory was the starting point for the initial researches of error correction algorithms. In the first decades of its development, it allowed to attract to the

theory of coding highly professional staff of the scientific community. And now the relay of leadership from the "classical" algebraic theory to the Optimization Theory (OT) noise-resistant coding in all applied aspects has been completely ended. The new stages of researches will include the increasingly broad areas of this branch of information theory.

The monograph presented powerful technologies of the decoding algorithms development for the four main classical channels of the theory of noise-tolerant coding with the minimum theoretically possible linear complexity at the code length and, at the same time (!), with an efficiency which is almost identical with the best possible optimal decoding (OD). Our MTD decoders in fact are as highly reliable as the OD. They provide the best error probability of error correcting techniques as OD with the full, exponentially complex with the code length total searching. Decisions of MTD decoders and other OT methods almost in the entire area of code rates R , lower than bandwidth of channels C , coincide with the optimal decoding. And the width of the region not yet available for the operation of the MTD decoders near the Shannon border is already quite small. Perhaps the size of this zone can be further slightly reduced in the future.

The efficiency and throughput of all our methods is confirmed by the interactive mutual work of the monograph with our world's largest portals on the coding theory www.mtdbest.iki.rssi.ru и www.mtdbest.ru . Dozens of demonstration modules placed at these portals and the best new software platforms for different types of decoders, mainly on the second one, illustrate high reliability and unique performance of our decoders, most of which (~ 40) are patented in Russia, abroad and else even in the USSR.

The characteristics of our algorithms, due to the current situation in applied coding theory issues, are not available to any other error correction methods with comparable complexity, and the difference between them, as far as we know, is very large.

The importance of balanced development of theoretical and experimental studies has been recognized in many branches of the world science approximately since 1985 and by the end of the Millennium has been actively and comprehensively taken into account in various investigations. The theory itself is always weak and very limited. But an experiment realized without theoretical support is almost always inaccurate or even just wrong. Our scientific school understood this interconnection in science somewhere in 1975. The correct balancing of the original theory ideas and sophisticated optimized modeling provided a Grand synergetic acceleration of our researches, which allowed us to solve the great Shannon problem.

In this regard, we also believe it is extremely important and absolutely necessary, solely for the disposal of the scientific community from fraud and falsification of scientific results in applied problems of coding theory in the next few years they should allow for publication and thesis defenses only those scientific works in which the authors have strictly and precisely described and analyzed decoding algorithms and,

necessarily, these algorithms working realizations. They should be implemented in C++ and allow full verification by experts on applied coding theory and methods of soft modeling these algorithms at high noise level. We are ready to agree that our demo program taken at the portal www.mtdbest.ru should be used to calibrate the throughput of the new algorithms proposed by the new authors (see hyperlink [demo_quick.zip](#) and a brief description of its user manual [directions](#)). This calibration demo program performs calculations at 8 threshold elements of the MTD decoder, equivalent to ~50 operations of summation of integers and 8 comparisons for each decoded data bit. Such a decoder at the usual PC under Windows with a very low relative levels of bit energy to noise ratio of the channel decodes with a high reliability digital streams at speeds up to tens Mbit/s. Small overheads of computational costs of C++ language and its other merits determine its very reasonable in this case requirements to the abilities of new algorithms authors to create their software implementation. This approach with the calibration of the efficiency parameters will allow, basically, to correctly assess the real levels of complexity and noise immunity of the proposed new decoders. Such a natural level of control the assessment correctness of the decoding methods basic properties will immediately put things in order in the applied scientific level of the development of all decoding algorithms. Only this approach, which is not rigid in any way, will allow to get rid of the boundless flow of fantasies, errors and outright deception in the research of applied issues in the field of error correcting codes.

We also agree that other algorithms for all types of channels, the parameters of which are specified in our monograph and in other publications of the scientific school OT, can also be used to evaluate various aspects of the new decoders effectiveness. We are always available and will be happy to advise the authors of new algorithms that will have high real characteristics.

There is no doubt that OT actively developing already ~5 decades will be sure to find else more innovative methods for creating efficient decoders and means for such projects, where the available technologies being near the bound of Shannon will not be enough. It should be clearly understood that the complex of coding systems design will continue to develop only with the progress of complex software systems optimization, and a powerful methods of searching global extremum of the functionals, which are actively and widely used in OT. Thus, the further movement of decoding technologies to the Shannon bound will be more rigidly connected with the development of computer technique and with special software than before. It's a natural way of real scientific-intensive technologies development. And this is probably the only constructive way. Our scientific school OT will continue to work hard to remain in the leader of theoretical and soft researches and developments the newest areas of our branch of science.

The crucial point that allows to consider the decoding problem for a large noise is completely solved, it turns out the complexity of MTD algorithms which with any complexity of design works on creation of the code and decoding algorithm is growing with the code length only linear, i.e. with the theoretically minimum possible rate. In

order to provide high reliability decoding remain available all those powerful techniques and principles that have built both classical coding theory and OT paradigms: different concatenation styles, divergence, BVA, DDCM methods and other features. Thus obtained a complete solution of the Shannon problem corresponds to the best possible levels of complexity, noise immunity at the input and reliability at the output of code systems. Of course, these parameters will be refined and improved over time.

And finally, the key point of the successful solution of complex problems of the digital world was a deep understanding by many experts that the most important circumstances that the achievement of seemingly impossible dreams of theorists in digital communication just methods of search global extremum of the functionals (SGEF) theories provided in the specific conditions of discrete mathematical spaces. No reasons to believe that there will be competitive other methods yet and most likely it will never be. Exactly theories SGEF allow us to consider only the methods of such a search with the complexity proportional to the code length. Other methods with such complexity are unknown. But this approach to the problem of decoding is operable only when both correctly solved the task of code selection with low EP level, which in the end, can be successfully used for finding the optimal decision with the best minimum probability of undetected error. Earlier this problem was solved in coding theory only the by total searching methods.

If we turn to the best methods of this class, we note with satisfaction that the patented by our team Viterbi algorithm for block codes (BVA) with the same complexity of decoders as for convolutional codes also completed the conversion of all applied problems of coding theory in the tasks of a combined global searching optimal decisions. BVA corresponds to the twice reduced exponent complexity of OD known from the "theorists" of block codes. Block VA completely eliminated from any competitions for block codes methods developed by algebraic theory. The grouping all types of VA and various MTD modifications in set of DDCM decoders, special algorithms only measuring the distance of their decisions to the received message, else more accurately indicates a set of methods, which, apparently, will solve all the problems of creating good decoders in all the above meanings. Most likely, all the new algorithms of past thirty years, unrelated to the theories of search for global extremum and precise distance measurement even with the assumption of further growth of their complexity, have no prospects of movement to the Shannon bound, nor, moreover, other possibilities of development. Of course, reality of the future time will strictly check the validity of such a harsh opinion.

Thus, OT took in the whole volume of development baton of applied methods from classical algebraic theory. This is not surprising, since algebraic theory has not solved any main basic problems of its development: it did not find simple ways of error correction above the level of half the code distance, it did not overcome the difficulties of decoding in a Gaussian channel, and it did not find linear with code length complexity of decoding. The classic theory did not find even decoding methods, with index in the exponent of complexity which decreases with increasing code length n .

Recall that for the MTD algorithms whose complexity is linear with code length, this index in fact is equal to the ratio $\{\ln(n)/n\}$, i.e., quickly goes to zero. In this group of DDCM methods solves all these problems of complexity such a naturally that even an exponentially complex VA, for example, in the schemes of the concatenated type, are in their short options very simple parts of code structures. In conventional circuits the characteristics of VA are dramatically better than for algebraic algorithms also. Let's not forget that already 20 years the technology allows us to create VA even for codes with a length of $K \sim 15$. So there are no questions about algebraic methods. A twice decrease in our patented BVA the exponent of its complexity compared to the estimates of the classical theory finally sets all decoding algorithms at their own places.

Thus set by Shannon the problem is solved in OT by making use at the different steps of decoder creation a powerful optimization procedures, including the search global extremum of functionals (SGEF). Of course, for the closer a project workspace of coding system to capacity the number of required operations of the decoder will be somewhat greater, that is understandable. The inevitable significant delay increase of the decision making is reasonable also..

Here are the main optimization procedure of designing OT algorithms. First, of course, it is actually the MTD algorithm configured on the approach with every change of the controlled characters to the optimal (wit exhaustive search !) decision with a minimum of its own complexity. This is a typical task of finding the global minimum of the functional in a discrete spaces.

The second optimization mean is a powerful tool used in creating effective MTD or DDCM algorithms are method for searching codes, in the greatest extent meting the criteria of errors propagation (EP) minimum when decoding. Completely unique theory was created for EP. It is not like 50 years ago very primitive attempts to describe this extremely complex process for the majority schemes. Optimization of codes on several criteria of EP improved multifold the MTD methods convergence to the optimal decisions.

Finally, the third and most difficult for usage global optimization procedures, there was a whole class of methods for the tuning elements of MTD decoders. This giant and difficult problem, exceeding in hundreds times the effort and time for both the first techniques required to develop effective ways to accelerate the processes of such tuning, which had been done quickly also. These works additionally has increased the rate of convergence of MTD decisions to the optimal ones.

The combination of these three basic approaches just became the nucleus of those methods that are in synergy brought the effectiveness of the MTD on complexity, noise immunity and reliability to a level that has long been inaccessible by any other methods of the classical algebraic coding theory.

Not superfluous in this connection to recall have long formed the opinion of mathematicians that the role of the optimization theories in mathematics is as great as the role of mathematics in science generally.

Thus, the transition to the creation of effective decoders only on the basis of the theories of global optimization of functional, delayed for many decades due to the usual in the sciences the influence of the human factor since the 80s of the last century, is completed. With the appearance OT the applied coding theory moving from classical algebraic preliminary stage to the wide application of powerful and fast optimization methods for decoding in all the technical systems providing with simple means and high reliability even when transmitted in a very noisy channels.

It is important to specify that a significant contribution to the OT development and its technologies bring new leading paradigm, which did not exist in this theory before at all: symbolic codes and decoders, divergence, parallel concatenation, parity check codes (PCC), BVA, convergent methods, fast erase recovery algorithms and rule DDCM, clearly recommending for the implementation certain methods of the OT. The most important role here belongs to the principle of divergence, which in certain techniques far advanced MTD algorithms in a region of large noise. This principle of non-concatenated increase the code distance of the applied codes have become a powerful design tool decoding methods of all types. It may be also indicated that parallel concatenation was first proposed by our team. Besides, convergent methods are difficult to implement outside of the MTD algorithms, for which, on the contrary, they are very comfortable. As for parity checks codes (PCC), this method has become a powerful tool for efficiently encoding only when MTD algorithms become really highly reliable and reaching decisions of the OD for relatively low signal-to-noise ratio. The decoders included in a concatenating scheme, implement best mutual work and improve their decisions for the total concatenated code. Concatenation in accordance with these principles for all classes DDCM algorithms provides an extremely simple solutions to all issues of high reliability decoding when a large noise level for the long time. . Simplified 3-D systems of this type also deserve careful study.

A wide range of potential opportunities and new directions for the development of decoding algorithms, briefly described in the last sections of the book also indicates completely limitless perspectives of OT, a new "quantum mechanics " in information theory.

We believe that it was possible to announce the change of leading ideology in applied coding theory approximately 15÷20 years ago when all important problems of efficiency have already been solved for symbolic codes and erasure restoring algorithms. At the same time for Gaussian channels MTD algorithms successfully worked with energy exceeding the level of the Shannon's bound at no more than 2 dB. However, the level of development of the OT ideology and the available computing technology did not allow us in those years to find optimization methods that would approach the workspace of MTD algorithms in Gaussian channel to a channel capacity at least to the level of the one and a half decibels. So we took then the unique right decision to improve further at that time characteristics of MTD decoders in Gaussian channels and only later to announce the total solving Shannon's task . Now quite feasible

MTD algorithms work at the energy-to-noise level, which is only at 26% higher than the power level of the absolutely elastic and in principle unattainable Shannon boundary. Such features are unavailable to any other decoding algorithms. This allows us to say right now about a complete change of leadership in all practical problems in coding theory to the new "quantum mechanics" of our scientific school, called Optimization Theory of error-correcting coding.

There is no doubt that a huge variety of possible schemes for global searching forms a large intellectual field to create a variety of methods to construct new codes, algorithms and technologies. . On their basis new talented researchers, designers and engineers will write the following pages in the OT and offer for our civilization the best technological and fast systems to ensure a high level of reliability in the transmission, storage, recovery and verification of data for the modern digital information world.