On the Prospects of Optimization Theory

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***Abstract*—the situation resulted from the solution of Shannon problem by optimization theory of error-correcting coding in the applied studies of coding theory is analyzed. Unresolved problems of classical (algebraic) theory are discussed. The results for main clusters of decoding algorithms built on the basis of Optimization Theory (OT) are considered. Further tasks for OT development are given, the methods for their solution are offered. Broad possibilities shown by new versions of Viterbi algorithm are mentioned. Advantages of main OT paradigms including the advantages of symbolic codes and divergent coding are shown. A special group of decoding algorithms with direct metrics control (DDMC) is singled out. The main ways to develop coding theory for the coming years and for the future are listed.**

Keywords—digital communication systems, data storage systems, error-correcting coding, block Viterbi algorithm, computer modeling, self-orthogonal codes, symbolic codes, concatenated codes, multithreshold decoders, divergent coding

I. INTRODUCTION

Successful solution of applied and theoretical problems shown by Russian science in the sphere of developing simple and highly efficient algorithms for error-correcting coding of digital data in the channels with noise proves the high potential of Russian informatics in a given sphere of knowledge. More than 600 blocks of reference and methodological data found on network resources of Space Research Institute of the Russian Academy of Sciences as well as Ryazan state radio engineering university [1, 3, 4, 6] reflect high dynamics of research made by the scientific school of Optimization Theory (OT) of coding: more than 400 works on this topic have been published so far [1]. The publication of a new monograph devoted to OT [7] required a detailed analysis concerning the current state of classical coding theory that will contribute to the choice of direction for further OT development. Specific characteristics for decoders of various types the description of which is not presented in a given paper due to their large volume can be found in [1–7].

II. THE STATE OF CLASSICAL CODING THEORY

70 years later since C. Shannon has published his historical article [8] that faced the science with the task to provide highly reliable communication via channels with noise the success in coding theory aimed at solving this task can be considered both significant and very limited. Its most significant results in the sphere of algebraic codes offered 60 years ago have become milestone achievements for binary symmetric channels (BSC) without memory though they turned out to be quite far from the possibilities of coding forecasted by Shannon. Therefore, it's not surprising that these coding having decoding complexity N being even smaller than $N \sim n^2$, where n – is the length of the code used (and complexity has promptly become one of the main criteria for the methods correcting errors being applicable) have led to the period of ubiquitous application of Viterbi algorithm (VA) the complexity of which has grown exponentially with the length K of coder, but error-correction was much higher even with short convolutional codes in Gaussian channels than with algebraic codes.

Since Reed-Solomon (RS) non-binary codes having proved their usefulness in many technical applications appeared, the following 60 years haven't offered any other non-binary codes with simple decoding and good error-correcting capability. In reality only short RS codes can be used till now leading to their inefficiency and operation in the conditions of low noise level. Nevertheless, the complexity of their decoders is quite considerable: $\sim n^2$. The best example for the practical application of algebraic codes is concatenated VA scheme with RS code. However, its advantages are determined mainly by the convolutional code used as well as by VA decoder.

As for algebraic theory, binary codes of this type did not manage to operate in Gaussian channels so easily as VA could and they did not correct errors beyond the minimum code distance d. Moreover, no method of their decoding the complexity of which increased together with increasing code length in linear way, i.e. with order $N \sim n$ has been found. All algebraic codes operated only in the area of channel error probabilities being quite low regarding the values that theory allowed. These facts are the reason for us to speak about the end of applied algebraic coding theory.

III. OVERCOMING THE FIRST CRISIS

High characteristics of VA algorithm reliability in binary channels have quickly led the decoding technique to the level being sufficient for various digital systems in the course of several decades. However, it turned out to be quite complex due to the necessity to remember all possible decisions the number of which grows with the exponentially growing code length. We should recall here that VA being an optimum decoder (OD) minimizes error probability of its decisions comparing the distances between the message received before all potential decisions were made and choosing the closest of them. This process requires large amount of memory.

The first successful attempt to overcome long-term large-scale crisis on coding theory had its aim at orienting on VA efficiency but not storing all decisions in a decoder, at choosing first some initial decision-hypothesis for a new algorithm. Later the decisions being closer to the vector received were offered to be used which, in some way, would lead to OD decisions. We should note here that after the author expressed half a century ago the attractive idea allowing us to decrease the complexity of such decoders up to linear one, proportional to $N{\sim}n$, i.e. up to minimum possible one it was implemented on practice only by OT scientific school.

These new algorithms for binary channels were called multithreshold decoders (MTD). All versions of MTD for block and convolutional codes had practically no difference from each other. It was 40 years ago already that in case of linear complexity they showed the highest validity and optimality of their decisions at noise levels absolutely unavailable for algebraic methods. Further the methods to search these codes were elaborated and their usage allowed bringing the validity of MTD decisions up to OD level even at coding rates being really close to Shannon limit.

IV. Non-binary and Other Code Problems Solution

After binary MTD were developed and patented it took another 8 years to realize the problem of simple non-binary codes decoding before the representatives of OT scientific school learned to decode special codes called symbolic ones with minimally possible, i.e. linear from code length, complexity. The results turned out to be so unusual that OT considered filing a series of patents for symbolic decoders [1, 3, 4, 6, 7]. Code lengths n and alphabet size q for symbolic codes unlike RS codes are absolutely independent making them suitable for many technical applications. The coincidence of decisions made by symbolic decoders with the results for optimum decoders even with high noise level as well as linear complexity of binary MTD have made these new algorithms the leader among non-binary methods. For 30 years already they still have no competitors in this field.

One more, quite a problematic aspect of coding theory is the impossibility to create VA for non-binary codes. Even for simplest one-byte, very short and thus ineffective code with code length K=5, having alphabet size q=256, the complexity of non-binary VA will be equal to $N{\sim}q^K=10^{12}$ that, surely, cannot be realized. Consequently, the critical importance of complete solution of the problem for optimum decoding of non-binary codes based on symbolic MTD with linear complexity becomes quite evident.

Here we should note that OT results were published also in English long ago but we still have no information about them being repeated. Such complete inaccessibility of OT results for other groups of researchers (Fig. 1) can be explained by the fact that the study of MTD and other algorithms required highly intelligent software of system style and highest level being typical for OT school. Besides, OT school has patented the simplest decoders with erasure channels having minimum complexity restoring erased symbols near channel capacity [1, 3, 4, 6, 7].

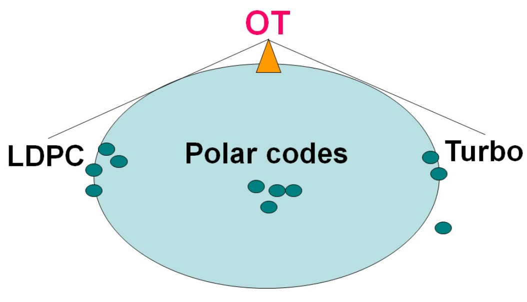

Fig. 1. Leadership of OT school

The fact that OT methods do not use the calculations with real numbers is also very important. This even more emphasizes the adaptability and efficiency of MTD algorithms becoming more important for the hardware implementation of decoders. The operation only with fixed point, i.e. with small integers, really speeds up the operation of software implemented algorithms.

Eventually, one more significant achievement of OT is the creation and patenting a special version of VA for block codes (BVA) with the complexity N, being equal to the most famous classical convolutional VA, i.e. with $N{\sim}2^K$, where K – is the length of coding register whereas until recently the representatives of theoretical school proposed BVA with $N{\sim}2^{2K}$ [9]. For comparison we can mention that VA with K=15 is known to be used [7] for one of NASA projects. Our BVA would be ~16000 times simpler for this code than similar OD for teaching students offered in [9].

V. Main OT results

The cornerstone of OT school is the Main Theorem of Multithreshold Decoding being simplest in form but full of deep system-philosophical meaning [1–3, 6, 7, 11]. According to this theorem, in case of linear complexity of decoding algorithm the decisions of MTD tend to OD decisions. OT immediately mentions that MTD is not OD and after a certain process to improve its decisions the algorithm can stop earlier, before it reaches optimum decision. During decades OT school created a lot of methods allowing to build the codes the usage of which gave MTD the possibility to reach OD decisions even at high level of noise for all classical models of communication channels. Nowadays the total number of such code clusters (typical combinations of code and channel parameters) has counted to 100 and their number is constantly growing.

All main OT results are based on the solution of optimization tasks of various types. We should mention that Shannon limit is fundamentally unattainable as it, like the speed of light for material bodies, is considered to be absolutely elastic. This statement leads us to the fact that channel capacity and MTD operating area always have a kind of gap between, and this gap is constantly decreasing due to the efforts made by the participants of OT school, as the experience shows. The situation with improving MTD characteristics (Fig. 2) demonstrates the direction of MTD performance to Shannon limit determining in this case limiting noise level theoretically allowing reliable data transmission for given parameters of code and channel. This work will be continued.

MTD characteristics mentioned are confirmed by the results from [1, 6, 7].

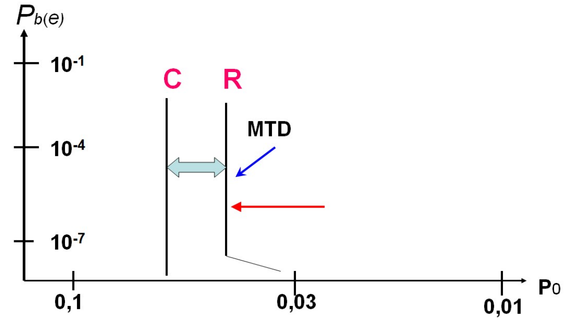

Fig. 2. Direction to Shannon limit

VI. MAIN OT TECHNOLOGIES

The development of applied coding methods allowed solving Shannon problem and making it not purely scientific problem but the one considered by the technologies based on the theories of global search. It is determined by the main MTD theorem proved in OT for all classical channels considered earlier in coding theory as well as sufficiently developed theory of error propagation in majority decoders. OT has created powerful technologies to build such majority codes that allowed MTD algorithm successfully operating near Shannon limit.

However, to receive actually high reliability of decisions in case of theoretically minimum possible MTD complexity and close proximity of code rate R to the capacity of digital channel C, i.e. when $R \lesssim C$, OT has created and successfully used dozens of methods being patented as well as software and hardware implemented [1, 3, 4, 6, 7]. They have formed the set of numerous simple techniques that improve MTD performance greatly. Further we shall list these techniques:

– divergent coding allowing nonconcatenated methods to increase coding distance of the codes applied in a consistent way [3, 7];

– group of decoders with direct metrics control (DDMC) having also advanced the characteristics of the best algorithms in the direction of even higher noise level;

– parallel concatenation, OT school being considered the discoverer of which [3, 7];

– new approaches to traditional schemes of sequential concatenation since the characteristics of MTD with optimum decoding fundamentally change the contribution to the overall efficiency from both codes entering concatenation system [1, 5–7].

We can name quite a number of optimization software platforms on OT portals [1] forming operation style of various MTD that create the area of custom parameters leading to the optimization of decoding experiments and the acceleration of these experiments being held. It is the formation of fundamentally new intellectual OT area based on specialized system software that created the conditions for the development of OT and the algorithms being sharply different from other methods. We should emphasize here that software development was held in strict accordance with the recommendations formulated and developed by OT.

VII. THE CORRELATION BETWEEN REAL AND IDEAL IN SCIENCE

The problem of correlating the possibilities of theory with the experiment escalated in 1980s due to the vast growth of computer science. Publications on this subject were presented on the portal of the Russian Academy of Science. Here is just one quote: «....confrontation between computer simulation and the theory based on mathematical methods is the disease of the century» [10]. The same diagnosis can be given also to classical coding theory.

Theorists of 1960s could use only quite simple formulae, e.g., binomial distribution that characterized the performance of rather inefficient algebraic decoders. However, the performance of VA and other methods including MTD could not be accurately calculated. Other specialists actively using computer simulation started to work with them. Theorists, on the other hand, remained "inside" algebraic theory where much could be "calculated in mind" or analytically. When simulation programs for students allowing them to make the simplest experiments in the field of coding theory appeared the necessity for wide application of full-scale computer simulation of all decoding algorithms was neglected. They were quite satisfied to use "code ensembles" performing sophisticated assessments for them. As a result, the process was suspended, new testable algorithms stopped to appear.

However, it was the awareness of the unity between theory and experiment by the followers of OT in 1975 that allowed this scientific school to develop a number of special software to study applied problems of coding theory. It was used to simulate different digital systems with developed processing methods as well as to set the parameters of codes and decoders. Such approach created powerful synergistic effect of accelerated research. Theory without experiments is weak but the experiment divorced from theory is almost useless and often wrong. As work [10] shows theorists often have erroneous conclusions. OT school makes use of many methods to control experiments based on even exotic but at the same time logic theory. Such crosschecking is quite productive and useful both for simulation and theory.

Therefore, the attention of OT school to software technologies on correctly chosen for this aim language C++ as well as high level of competences shown by its members as programmers – all this taken together allowed developing original software systems of codec design.

VIII. CURRENT DEVELOPMENT LEVEL OF OTHER DIRECTIONS IN CODING THEORY

It should be noted that a large group of scientists that belongs to theoretical school applying no simulation and having poor knowledge of modern programming methods can experience difficulties while making complete models of decoders and evaluating their reliability, error-correction and complexity. The work in «pure» coding theory, in our opinion, has not produced any single specific decoder for the last decades. We wish the motto «Programming is the second literacy» was relevant again. The algorithms for hundreds of «code ensembles», successfully defended as doctoral dissertations back in 1990s remain fiction as decoders can be created only for specific codes.

Nowadays many publications announce the significant results for absolutely inefficient and complex, in our opinion, decoders with complexity $N{\sim}n^2$, when back in 1981 the data about MTD-algorithms with complexity $N{\sim}n$ and OD characteristics were published [11, 12], being mentioned 15 years ago in a relevant coding reference [6]. For the last 30 years the whole series of doctoral dissertations have announced 3-10 times reduction (in comparison with exhaustive search) of complexity exponent to decode some codes [9]. But in practice it always corresponds even to quite short codes, e.g. with $n \lesssim 10^4$ complexity of decoder N exceeding the number of atoms in the Universe. Surprisingly, the aforementioned BVA with double exponent complexity is still discussed in the manuals for student learning [9].

Polar codes, to our mind, have also had no help for classical coding theory to get out of crisis [13]. During the last decade no complete set of real characteristics of such decoders was shown. The majority of the decoding algorithms for polar codes described for example in [14, 15] have complexity $N{\sim}n^2$ or even higher than $N{\sim}n^3$. These algorithms are too complex for long codes.

Finally, we should note here that after 30 years that complete OT theory has been published together with the reference book [6] the students in all Russian universities including the most prestigious ones are mainly taught using algebraic books and course programs that sometimes have no information about Viterbi algorithm. This situation urgently needs to be changed.

IX. General Situation in Coding Theory

Currently, applied OT has considered all main coding methods suitable for broad use. They operate in close vicinity with Shannon limit with MTD algorithm linear complexity achieving optimum decoder decisions. Theoretically MTD decoders demonstrate maximum possible rates of hardware decoding as, e.g. MTD decoder for convolutional code (Fig. 3) developed in Space Research Institute of the Russian Academy of Sciences on the basis of FPGA ALTERA – its rate being about 2 Gb/s [2, 3, 7]. Software MTD versions operate at the rates from tens of kilobits per second up to megabit per second [1, 3, 4, 7]. It becomes clear that the delays in the decisions of algorithms operating in the vicinity of Shannon limit are quite large which is natural when using long codes.

Fig. 3. Multithreshol decoder for flash-memory, space and optical channels increasing their efficiency by 3-10 times

Besides, OT is the basis for fast decoders for optical channels having no analogues so far [1, 3, 4, 7]. Several extremely simple decoding algorithms with OD characteristics for flash-memory have been developed, this fact required to increase reliability of decoding results by several decimal orders [1, 3, 7].

X. Conclusion

Monographs [2, 3, 7] and resources [1] widely present Optimization Theory in different formats having completely solved, in authors opinion, Shannon's problem – the task of fast highly reliable decoding near digital channels capacity. Other decoding algorithms with confirmed recognized characteristics comparable with OT methods received in the whole set of parameters "reliability – error-correction – complexity" are absent.

Materials in OT subject published in sources [1] (Fig. 4), attract the attention of specialists throughout the world. In 2015, e.g. they were visited by more than 100 thousand readers from 87 countries that testifies to the authority of Russian scientific OT school.

Further progress of OT will be connected with various systems of multi position signals [2, 4, 7], as well as with more complex communication systems and networks. The scientific school of OT is ready to provide full support for prospective developments made by other organizations and initiative specialists who wish to develop simple, highly efficient decoding algorithms of various classes and solve other important applied tasks in adjacent directions.

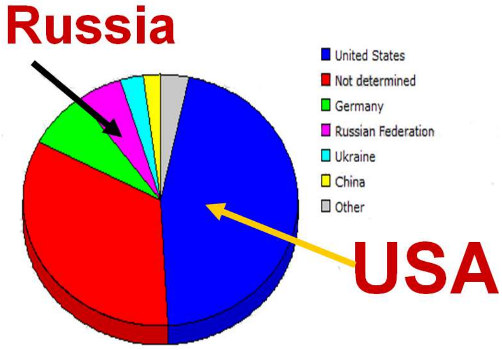

Fig. 4. The popularity of portals devoted to coding methods www.mtdbest.iki.rssi.ru и www.mtdbest.ru

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