N.A. Kuznetsov, V.V. Zolotarev, Yu.B. Zubarev,
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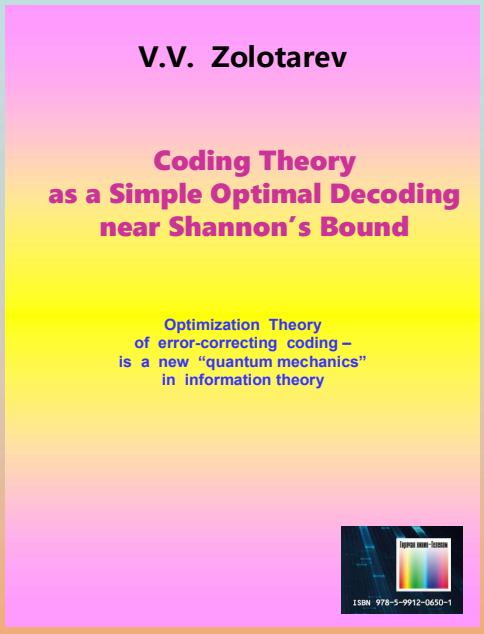
Problems and Discoveries of the Optimization Theory for Coding near Shannon's Bound

(OT in illustrations)



Moscow
SRI RAS, RSREU
2020

UDC 621.391.15

Authors: N.A. Kuznetsov, V.V. Zolotarev, Yu.B. Zubarev, G.V. Ovechkin, R.R. Nazirov, S.V. Averin.

K-91 Problems and Discoveries of the Optimization Theory for Coding near Shannon's Bound (OT in illustrations)
Moscow: Hot Line - Telecom, 2020. - 45 p.: Il. ISBN 978-5-9912-0872-7.

Kuznetsov Nikolay Alexandrovich — academician of RAS, doctor of technical sciences, professor, kuznetsov@cplire.ru .
Zolotarev Valery Vladimirovich — leading researcher, professor, doctor of technical sciences, zolotasd@yandex.ru .
Zubarev Yuriy Borisovich — member-correspondent of RAS, professor, doctor of technical sciences .
Ovechkin Gennady Vladimirovich — professor doctor of technical sciences, g_ovechkin@mail.ru .
Nazirov Ravil Ravilyevich — head of department, doctor of technical sciences, rnazirov@cosmos.ru .
Averin Sergey Vladimirovich — Deputy Director, doctor of technical sciences, ser-averin@yandex.ru .

The theoretical and applied results of the modern noiseproof coding theory are summarized in a popular form, using extensive illustrative material as a problem of searching for the global extremum of the functional in discrete spaces.

The authors propose the simplest algorithms for decoding of all channels classes with independent character distortions and with a linear of the code length complexity. The decoding error probabilities correspond to the best possible veracity levels, which is provided usually only by optimal full searching error correction methods in noisy channels. It is extremely important also that these highest characteristics of algorithms developed by Optimization Theory (OT) can be achieved even very close to the Shannon's bound.

For many specialists in the field of communication systems, engineers, undergraduates, as well as graduate students of mathematical and radio engineering faculties.

-=-=-=-

В популярной форме с использованием обширного иллюстративного материала кратко изложены теоретические и прикладные результаты современной теории помехоустойчивого кодирования как задачи поиска глобального экстремума функционала в дискретных пространствах.

Авторы предлагают простейшие, с линейной от длины кодов сложностью реализации алгоритмы декодирования всех классов каналов с независимыми искажениями символов. При этом вероятности ошибки декодирования соответствуют наилучшим возможным уровням достоверности, которые обычно обеспечивают только оптимальные переборные методы коррекции ошибок в шумящих каналах. Крайне важно, что эти высокие характеристики алгоритмов, разрабатываемых Оптимизационной Теорией, могут быть достигнуты даже в непосредственной близости от границы Шеннона.

Для широкого круга специалистов в области систем связи, инженеров, студентов старших курсов, а также аспирантов математических и радиотехнических факультетов.

UDC 621.391.15
ISBN 978-5-9912-0872-7.
Published by Hot Line – Telecom, Moscow, 2020 .
www.techbook.ru . **Electronic edition.** № 180893, at 30.03.2020.
Without any support. For genuine education. Free of charge.

To our readers

Russian and our foreign readers with interests in digital signal processing are welcome to get this very informative gift, a kind of comic about the extremely problematic area of computer science - applied error correcting coding theory. Especially valuable, that this comic was created by participants in a Russian scientific school Optimization Theory (OT), developing, moreover, extremely successful the main issues of creating decoding algorithms in noisy channels in the conditions of the maximum permissible noise level, i.e., directly actually near the Shannon's bound.

This subject has been studied by million of professionals in all technologically advanced countries over the past 70 years. The authors deal just with those problems of applied coding theory, that really have not been solved for a long time, persistently hushed up or extremely wrongly evaluated. To them relate complexity, feasibility, and even the main outcome criteria quality of decoding algorithms - the probability of an error at the output. Here it is very important, that there are many monographs, review publications and reference books at the end of this booklet, strictly confirming all those results and relationships in applied issues of coding theory, that are quite simply and clearly described by the authors of the booklet.

It is significant, that many difficult and even extremely problematic for many network developers questions, have been solved by the scientific school OT. This, for example, is arbitrarily high reliability of digital data storage, extremely low power of data communications in noisy channels near Shannon's bound or exceptionally simple optimal decoding (OD!) of block codes, that for the previous already completely non-classical now theory it remained unsolved and a very difficult task. And involved in the Internet of things people, of course, are surprised to learn that the OT methods, in particular, multithreshold decoders (MTD) will generally use at several orders (!) less energy of autonomous power supplies simply because they as many times faster than other equivalent at fidelity methods, and besides, they also provide almost always the optimal, most reliable decision. It is as important as the operation of MTD decoders or block Viterbi algorithms (BVA), patented by the school of OT, near channel capacity, i.e. at the lowest possible power of transmitter signal, which also greatly saves battery power. Such there are a lot of useful properties of decoders offered in OT, and all of them turn out to be very useful for a wide range of code applications.

It would remind to our readers that specialist's reorientation from previous methods to OT technologies problem does not exist. MTD methods are the simplest well-known majority schemes, the correction processes in which are very clear, but repeated several times for special codes, which are also easy to build. And MTD decoders are so simple that even with repeated correction iterations they are still at several orders (!) simpler and faster then others. Their limiting characteristics are determined by binomial or other similar distributions with which every engineer is familiar. But the probabilities of error of any specific decoding algorithms, and not only in OT (!), can be obtained only by the full simulation of decoding processes. Scientific school OT has understood it 50 years ago that, together with other original results had allowed it to become a world leader in algorithm development decoding. OT interacts very closely with a subtle, sophisticated experimental modeling, which gives them both a very powerful synergistic acceleration.

Finally, we point out that the "quantum mechanics" of information theory, as the scientific school OT calls its theory, is extremely compact, which emphasizes nods its perfection. It contains a very small number of formulas, but includes all the necessary mathematical expressions when describing OT possibilities for all classical channels considering in the coding theory . This greatly facilitates the reading of all OT monographs that are strict, logical and understandable systemic-philosophical treatises in the applied information theory.

Absolutely grandiose role is of undoubtedly the largest in the world network portals of the scientific school OT. They can find there hundreds of books and reviews on OT, computer cartoons by codes, as well as software platforms, which, after reading this booklet, you can almost immediately use to study OT, MTD and various types of VA , as well as for diverse studies in this interesting field in which Russian scientific school OT overtook the whole world for several decades it is thanks to the unique software systems created by her for research and development in applied coding theory.

The booklet on codes offered to the foreign readers is actually outlined provides a history of the development of the main branch of the applied theory of coding in RF and in the world by version of the scientific school OT. It very unexpectedly coincided in form with comics that are popular now with readers and publishers, giving it a special useful originality. I am sure that this booklet will accelerate development theory of error correcting coding and prepare a new generation coding technology specialists who will ensure conservation world leadership of the domestic school of OT and its further rapid development based on the most modern scientific theories and unique innovative computer technology.

Academician of the Russian Academy of Sciences N.A. Kuznetsov

Introduction to New Coding Theory

The fate of leaders

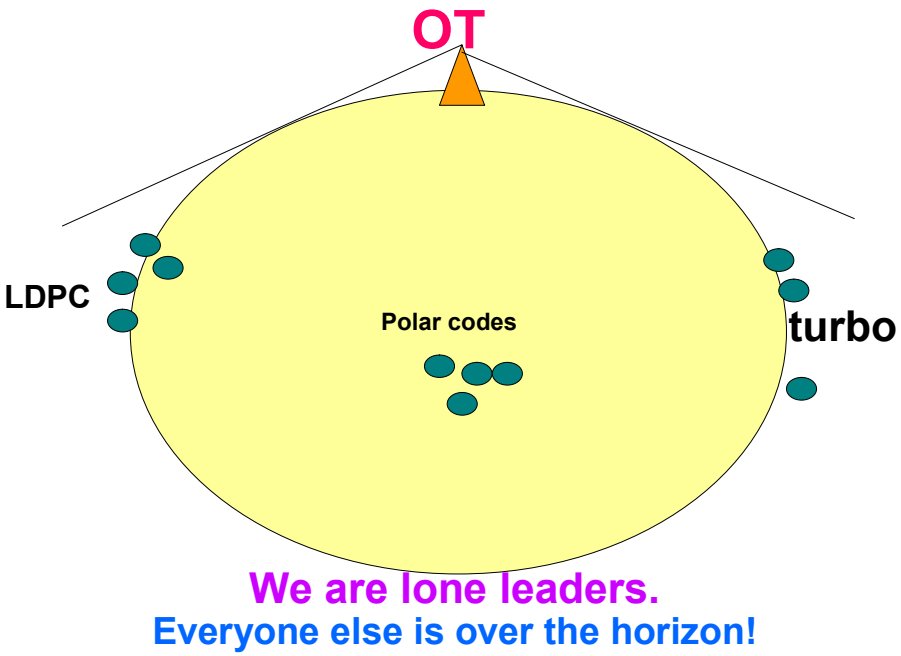


Fig.1.

Our picture book appeared as a result of intensive work during many decades the scientific school of **Optimization Theory (OT)** of error correcting coding over the problem of simple and efficient decoding near the channel capacity that staged 70 years ago by great Claude Shannon.

We have solved this grand task!

The first picture in this book shows our loneliness "on the top of coding theory world". We are very sorry for the lack of colleagues with we can discuss the results of OT. These results are **very far ahead of all**.

We suggest you familiarize yourself with our OT, so to say in the “comics” format, i.e. very briefly. But a minimum of references to serious work we give at the end of this booklet. These books should be read carefully and, as we have seen from communicating experience with colleagues, even several times. The reason for this is simply **that our turns in research** aimed at solving Shannon’s problems were simple, very logical, but **often very unusual**.

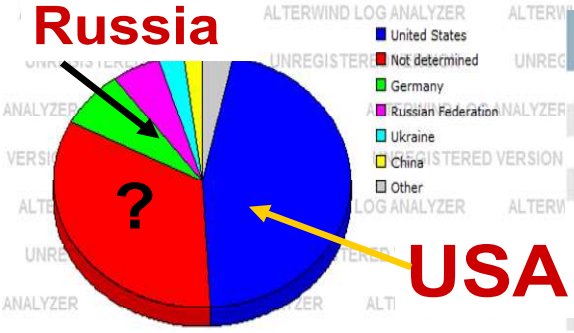
So read, be surprised, doubt, agree (**or not!**) and join the research in OT. **Be the real science leaders!**

OUR portals on OT and MTD

www.mtdbest.ru

www.mtdbest.iki.rssi.ru

For 2016 - more than 105 thousands readers on our portals from 94 countries of the world



Rank	Country	Visitors	
1	United States	13988	45.79%
2	Not determined	10280	33.65%
3	Germany	2178	07.13%
4	Russian Federation	1607	05.26%
5	Ukraine	859	02.81%
6	China	619	02.03%
7	United Kingdom	228	00.75%
8	Kazakhstan	103	00.34%
9	Belarus	80	00.26%
10	Italy	52	00.17%

Fig.2.

The key figure for the correct perception of the development situation with coding theory in our country and in the world is **this diagram**. It shows that **the whole world** understands a lot in OT now, though not everything. However, this statement of the problem turned out to be absolutely alien to so called a "classical" groups of coding theory specialists.

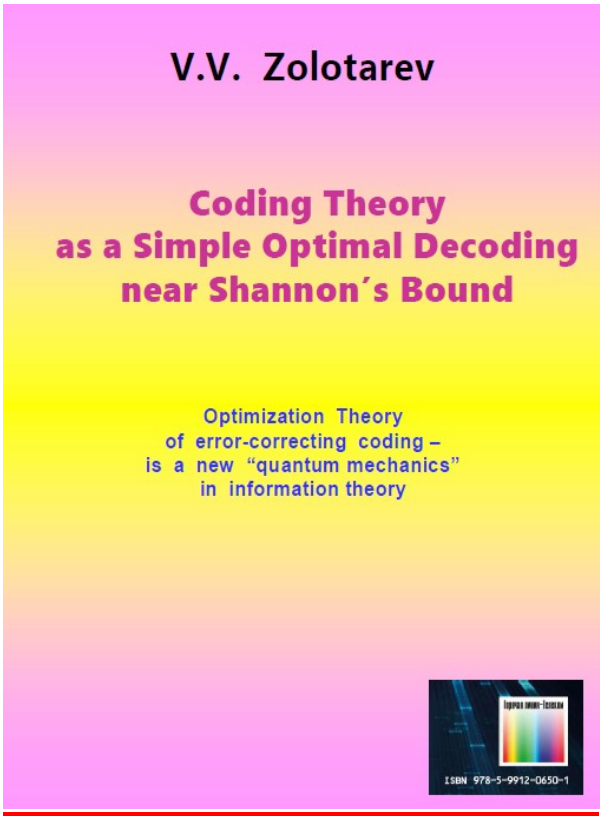
We deeply regret that our contradiction with the bulk of the experts who wrote various articles on the coding theory - it is just that in the early 70s we realized that **the coding theory is not a mathematical problem at all!**

Its problems are solved only in the union of theory and highly professional modeling, in particular with C ++ language usage. **No error probabilities of any algorithms** at high noise levels can be obtained analytically**.** This is a fundamental principle! Reliability assessment is always only a modeling task.

The development of our theory has always been guided, mainly, by the results of subtle experiments. And the simulation, in turn, showed those areas, which theory should focus its attention on, sophisticated but always simple calculations. This is a synergistic mutual acceleration **has solved all the coding theory problems.**

Yes, OT is mastered faster in foreign countries than in the homeland, in Russia, as can be seen from the diagram. And the "classic" theory, which has not really become in Russia as such, left the stage several decades ago. **Its successes are zero!**

The world has seen the novelty of OT and is mastering it. We believe that our young scientific generation going into coding theory area would find a true balance between theory and an experiment as well. We are sure that it will be so, although the slogan of this century beginning that "**programming is a second literacy**," alas, has long been disregarded. **It's a pity!**



Pic. 3

To our readers who wanted to read something about multithreshold decoders (MTD) for a long time, we are pleased to announce that **in 2018 our new monograph has been published,** which you can see in pic. 3. It contains all the necessary information to understand the idea and details of such decoders. Total interaction of this monograph (as well as previous ones!) with our web-portals, which are presented on pic.2, containing in total more than 600 blocks of information, references and scientific or methodological data, makes **the real volume of the book about 5 ÷ 10 times greater its physical size.** This allows consider that students, engineers and researchers of Russia and in the world have everything real opportunities to understand the essence and the many nuances of Optimization Theory. **Dozens of software platforms on these portals** for the various MTD algorithms versions development allow quickly start independent researches in the OT field.

The scientific editor of this monograph is Academician of the RAS N.A. Kuznetsov. He led main academic institute during a very long time, that was the leading one in error correcting coding sphere - IITP RAS. He believes that "some of the author's results are discoveries in coding theory". And in his introductory article to our monograph he wrote: ".... 2018 is a jubilee year for the coding theory. 70 years ago, Claude Shannon put forward the problem of simple and effective decoding before science and technology in its remarkable article "Mathematical Theory of Communication". It is really nice to find its successful solution in the anniversary year **in the monograph of the Russian scientist**".

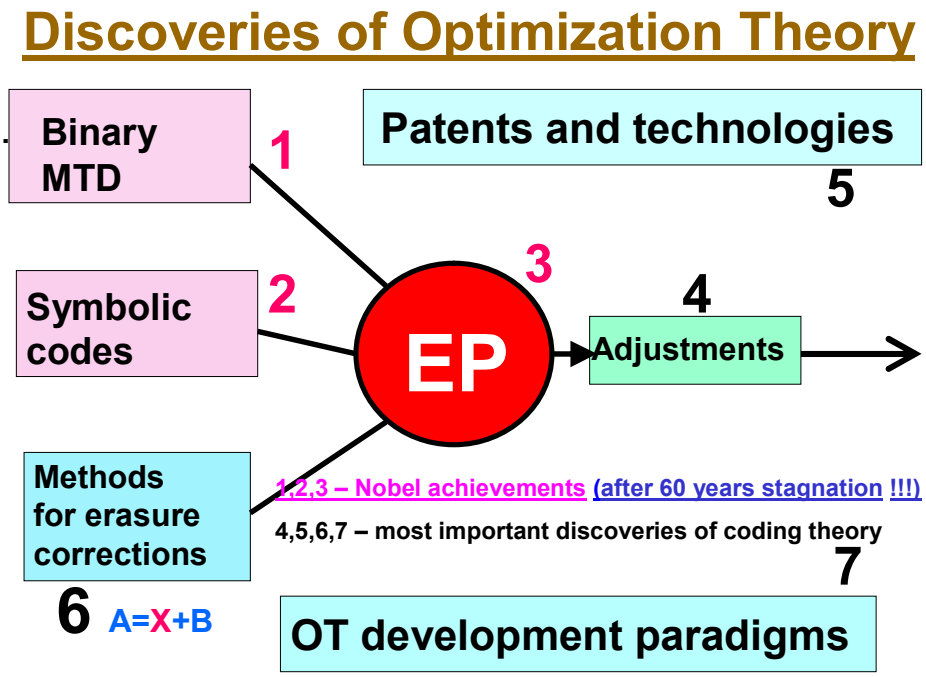


Fig.4.

We suggest that you really understand and agree that **the previous coding theory**, that could be called "**classical**" until recently, **completed its existence.** The reason for this is that during the last decades it has not presented any single concrete decoding algorithm in conclusive and well proved format. In fact, **besides** of the scientific school **OT results**, you can find nowhere any particular decoder, that would be published in the form of an article with a sufficiently large amount of digital material and which at the same time would be accessible to everyone in the form of program, preferably in C ++ language, which would be an operation model of this decoder in a channel with a sufficiently high noise level. This is the only way we can evaluate all the features of the decoding algorithm in a single stringent criterion **"noiseproofness-complexity-veracity".**

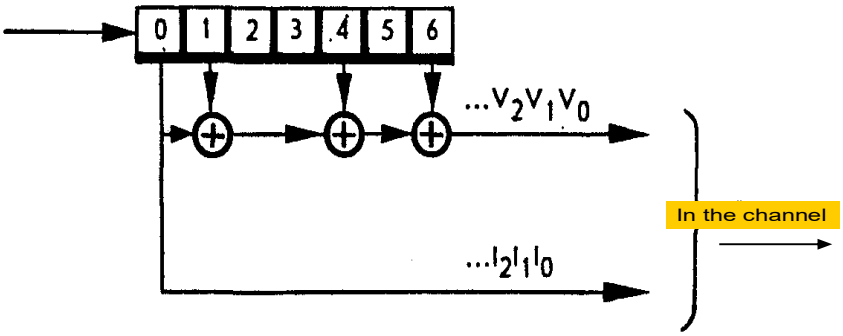
Therefore, to convince out our dear readers that we suggest a real complete simple and technological theory, i.e. to interest all "new Newtons" in the OT development, pic.4 shows the block diagram of the OT structure. It consists of several large clusters (sets of typical parameters of codes, decoders and channels), three of which we and some of our colleagues **consider results of high Nobel level,** and four others - **most important discoveries in OT**.

We also note that the **OT is extremely compact,** set out as complete systemic-philosophical treatise, but contains all the necessary formulas for MTD decoders development, BVA and OT. Let's see what our OT can do in terms of creating simple decoders for all traditional models of communication channels.

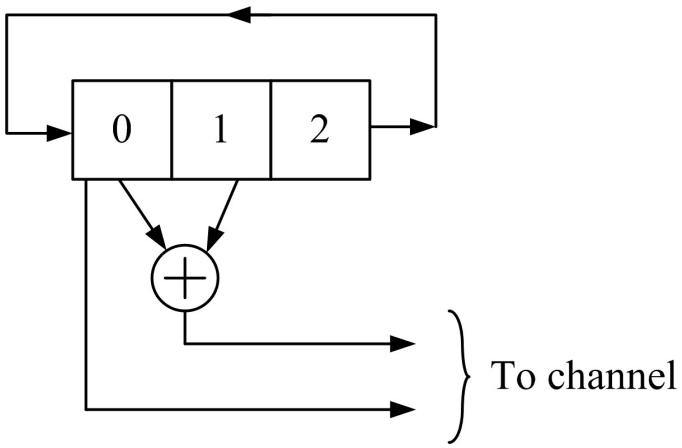
Why OT is about the best algorithms

A) - The simplest encoders. Encoding is easier !!!

An example of coder for convolutional code with code rate R=1/2 and d=5



B) Coder for block code

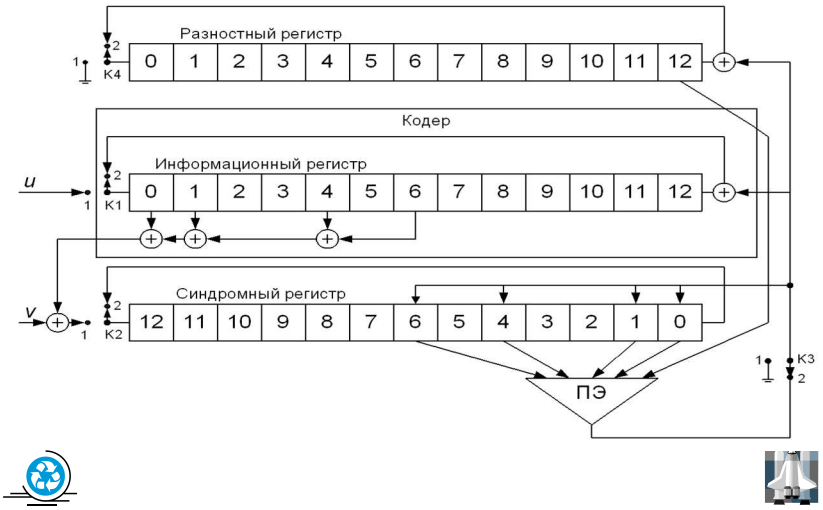


In fact, the **complexity N of encoders** of majoritarian decoded codes - is the **smallest possible**: N ~ n, where n - is the length of the applied codes, which can be easily seen at the slides presented. It's simpler not to come up with anything. Do not forget at the same time that many codes that for some reason (absolutely in vain!) they consider "effective", have encoders with complexity of the order of N~n*ln(n). In fact, if encoders are complex, then decoders are also almost always more complicated than what is really possible even with comparable efficiency.

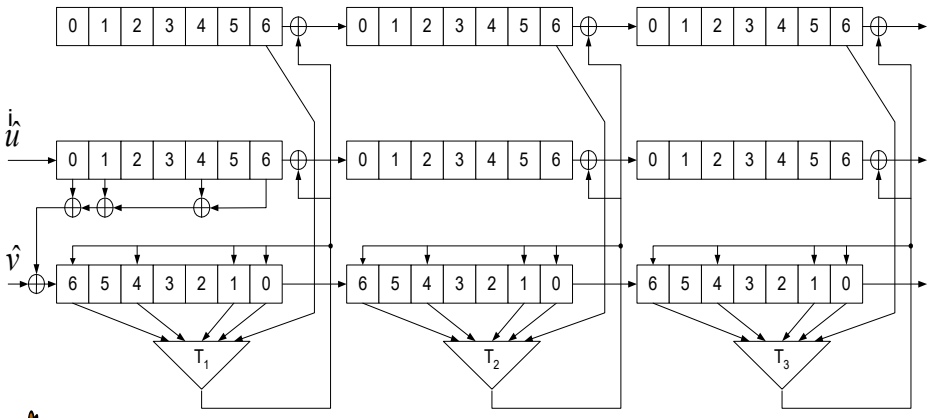
So we have found the first reason for the **majoritarian (threshold) decoding algorithms advantage.** It is quite useful, but so far **not the most important one**.

b) - simple decoders

Block multithreshold decoder for code with R=1/2, d=5 and I iterations



Convolutional multithreshold decoder for code with R=1/2, d=5 and 3 iterations



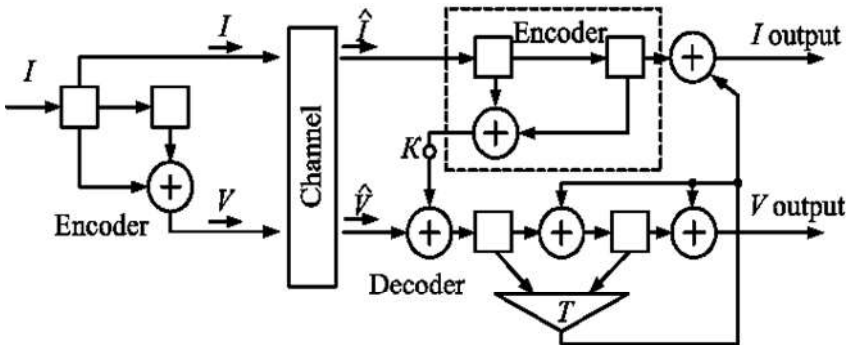
→ It is a foundation of Optimization Theory

Fig.XXI

But simple **majoritarian decoders (MD)** are also a fact. All other decoders of even "the simplest type", no one can even draw on paper so that it is clear. And again: all other decoders work with real numbers, but our majoritarian ones - only add up small integers . And the formal **complexity of majoritarian decoders** (the number of the simplest operations of addition of small integers) N ~ n, i.e. is also minimal. **The others have much more N**. This is already a more **important reason**.

The most important entity: MD - can be optimal decoders (OD)!

Revolutionary interpretation of the linear code syndrome meaning



Pic.**A**

The strange title seems to be not from exact scientific, isn't it?

Let 's remember, that OD measures the distance of the received vector to all potential decisions and chooses from this **exponentially with n growing list** such a decision that is closest to this received vector. So **Viterbi algorithm (VA)** works - **salvation for applied coding theory** for almost half a century, although it is exponentially complex: it needs to remember all its possible decisions.

But the coding system at pic.**A** is not at all similar to the VA scheme. Here is MD for the convolutional code. And nevertheless it became the starting point for OT. Why? It is because that no one has ever in the last 60 years of life in the coding theory have noticed the following: VA measures the distance from the received vector to possible decisions. So **this MD does the same! But!** Here the threshold element in MD based on the syndrome vector measures this distance only with syndrome characters using. Nevertheless it is necessary to include in these measurements information bits also, as in VA. O'kay. But what for? Yes here, if you measure the total distance, then (if you're lucky!) MD will begin to correct errors better, because VA does it perfectly!.

And it turned out that - yes! The decoder at fig. **XXI** had got one more register, which marks the decisions of the threshold element (TE) in the MD decoder. And suddenly **(!)** it turned out here that at every step, when information characters are changed such MD, which is called a multithreshold decoder (MTD), strictly approaches the decision of OD. **Doubtless, it is a super-revolution!** Because that the **complexity** of the MTD increases **only linear with the length *n*** of the code, but it replaces exponentially complex OD.

And no one else can do this!

Main theorem about multithreshold decoding

- At each moment of information characters inversion MTD decisions strictly approach the received message, so the likelihood of MTD decisions is strictly increasing.

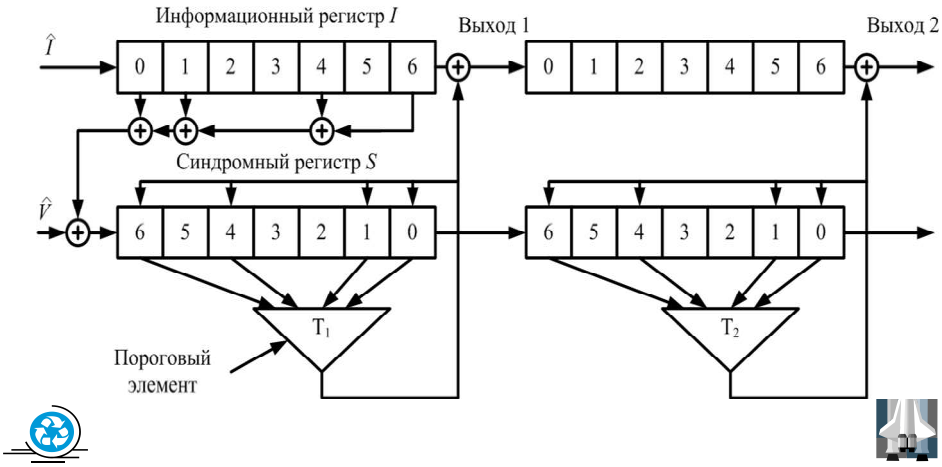
Consequence-goal

- MTD can achieve at linear complexity of decoding with code length the most plausible decision which usually requires exponential with the code length of the complexity .

On the slide the **OT Main Theorem** is presented, many years ago defined its prospects. But in the consequence it was immediately indicated that the decoder just **can come** to OD decision, **but it has not to do it at all**. Moreover, see, please: MTD - it's just re-decoding! That is all? So repeating decoding ineffective, that really appeared out more than 50 years ago in various large-scaled studies. And here authors are praising MTD, only because it really extremely simple? What's the matter? Does it works well? What has changed in decoder ideology, if **MTD - a panacea**? Below as a reference we can see a slide with the usual repeating decoder. It is not really much different from MTD.

What's the matter?

Threshold redecoding of convolutional code R = 1/2, d = 5 and n_A = 14. This is weak!



Well, let's compare the last slide with the MTD decoder.

It has not differential register, which contains decisions of threshold elements (TE). So for this scheme it is impossible to prove the Main Theorem. So what else?

But the second reason was beyond the boundaries of decoding schemes.

HERE IT IS!!!

Errors propagation

- **The problem of evaluating for errors propagation (EP) have been solved for the majoritarian decoding.**
- **Methods for assessing EP have been created for different types of codes**
- **Software complexes have been written for searching codes with a low EP level effect.**

Many billions of known codes for threshold (majority) decoding are such, in fact, that after the first decoder error the next errors form packets. But MD and MTD are both usually designed, of course, for the case of independent errors, for example, binary symmetric channel (BSC). Means the similar **packets are too hard** for such MTD. But we noticed long ago that if you build the MD code, which has the same code distance d, for example, $d = 11$, as the most short of possible ones, but special, which will be **in 5 to 10 times longer**, then re-decoding for this code will be quite successful. Moreover, sometimes the third correction attempt will be very helpful as well. Characteristics of such codes described in all our monographs. And for them there is again a differential register in MD, which in the MTD is very useful. In this option the **Main Theorem** starting again "to play" for MTD. And then we need to look for conditions when you can no dream longer, but hope for the reality of achieving **OD decisions on MTD base**. How?

Based on **the theory of errors propagation (EP)!**

We have formulated the problem, solved it and, most importantly, based on this theory have found such codes, of course, very long, which allow achieving OD decisions even with a very high noise level, literally close to the **Shannon's bound**.

Over the past 50 years, the EP problem has not been solved in any form by any scientific team in the world. We realized that the "**golden key**" of the optimal decoding at a high noise level is hidden in **the EP theory**, which allowed find then the necessary codes. Theory of EP and its applied results after the discovery of MTD decoders turned out to be **the second achievement of undoubtedly the highest Nobel level**. These new codes work near the Shannon's bound. But so far there were only **BSC type channels** in our scientific story.

However, BSC is only the first touchstone when comparing codes. The whole world likes to compare decoding algorithms for Gaussian binary channels, when quantization of a binary stream is used in receiver, and the decoder takes full advantage of the capabilities of such a channel. And **2 dB** additional energy of coding gain (CG) in this case is always very useful and important. It was the reason to transfer the contest of decoders at the Gaussian channels.

Well, and how is MTD doing there? Excellent! Like VA, **MTD for Gaussian channels** also receives these **2 dB** of additional CG guaranteed by the theory. So moreover, we have patented block VA (BVA)!

Block Viterbi Algorithm, patent RF


ПАТЕНТ

НА ИЗОБРЕТЕНИЕ

№ 2608872

Способ кодирования и декодирования блокового кода с использованием алгоритма Витерби

Патентообладатель: *Золотарев Валерий Владимирович (RU)*

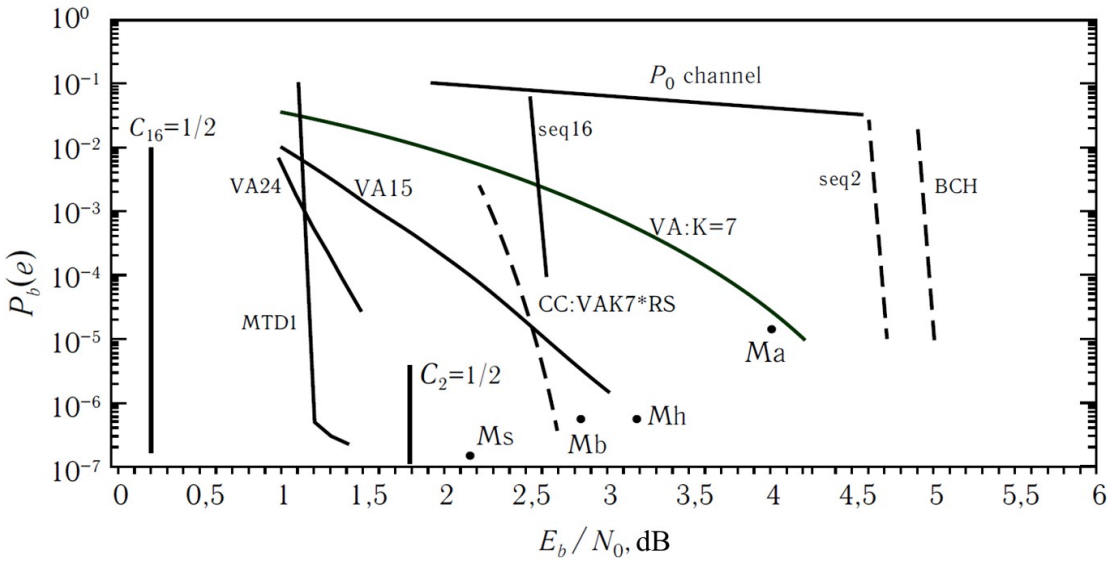
Авторы: *Золотарев Валерий Владимирович (RU),*
Овечкин Павел Владимирович (RU)

Recall that the most important decoding method that **saved the whole applied coding theory for about 50 years - Viterbi algorithm (VA)** for binary convolutional codes, and literally everyone wanted to convert it during a very long time in such a way, that it would be applicable to block codes. As we know, the complexity of VA - exponential function of the length **K** of the coding register. And over the past 30 years such “games” with OD for block codes, partly based on VA, really allowed the large "scientific" group to defend doctoral (!) dissertations in many countries. These very “creative” individuals described OD for block codes, but with decoder complexity, which is proportional to the exponent of **2K**, i.e., catastrophically huge. But we have a **patent for block VA (BVA)** with the **same complexity as for the convolutional VA**, i.e., with the exponent of K, but without a "deuce" near **K**, like that of homegrown "inventors".

But the double exponent - is it a lot? Last century NASA did VA for the convolutional code with K = 15. So if for the same task of NASA to build block VA, its complexity in the implementation of the "new Doctors” methods will be in ~ 16'000 times more than our patented BVA complexity. So all versions of **VA and MTD** form an extremely effective and technological pool of methods, and **nothing better is possible to invent!**

But what can real MTDs do?

OT: receiving the baton from the algebraic theory



There is no doubt that each graph with specific characteristics algorithms deserves multi-page analysis, and sometimes several paragraphs in the mono-graph. Therefore, a detailed and, most importantly, comparative analysis we are not going to do now. Our goal is to give a general, but **short and correct review of the whole situation in theory and coding technologies**. The deep analysis was done in our monographs, reviews and articles on OT. We are kindly welcome to read it!

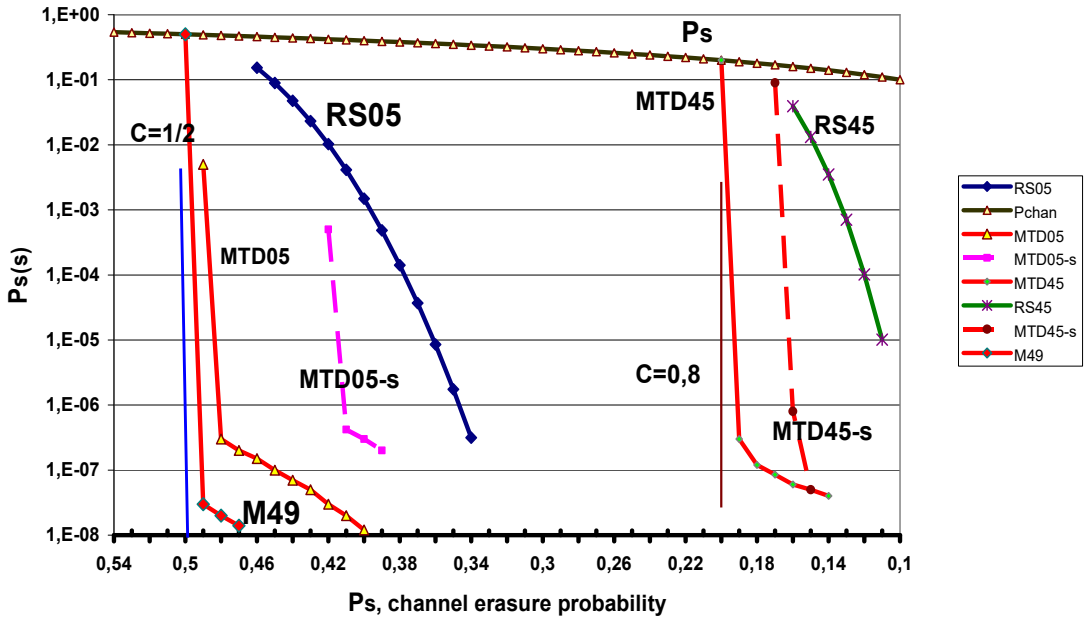
Here we will consider the presented slide in the proposed style. On it in the traditional format, the capabilities of the MTD and VA methods are depicted: Shannon's bound in the BSC and Gaussian channels, and also results for sequential algorithms and BCH codes. Total result: the "old" methods are noticeably weaker and slower **MTD decoders, which in addition have the lowest possible complexity theoretically linear.** Moreover, points "M" with indices, also related to MTD decoders, were obtained many decades ago, and now OT possibilities are much wider. For example, **point Ma is the MTD result for BSC** with 11 iterations, published after a lot of problems still **in a very distant 1986**, corresponds to the level of OD for a very small ratio of E_b/N_0.

So in those years, as well as in this millennium, such a grandiose fact "theoreticians" managed even not to notice. They did not begin at all to analyze such **extremely impressive** for that time MTD **results.** But in the end, they stayed by now in the same 80s years of the last century. Neither then nor now they had ways for this situation analysis. They have not understood that **at high noise levels for effective decoders there are no analytical methods for accurate determining the probability of error correction.** This can be done, and very quickly, **but only at software models**. But they still cannot do it. Nobody will do it in the future as well. It is impossible. So it became the **full end of the "classic" methods three decades ago!**

Erasure Correction Methods

Intuitively clear that to correct accidentally erased characters in the erasure channels is much simpler than errors in unknown places. However it does not help to "classical" methods to significantly simplify them.

Characteristics of MTD and RS codes in the erasure channels



This slide, if referenced briefly, shows the wonderful possibilities of MTD in the erasing channels with the probability of erasure in the channel**, only at 2% less** than with equal code rate R and channel capacity C . This is an inaccessible level of proximity of R and C for another methods. And the MTD complexity is absolutely negligible: at each iteration decoder must **solve the simplest equation for three small integers: A = X + B.** **That's it!**

The complexity of other methods to restore erased characters is much more than N ~ n. This allowed some authors to write that **complexity MTD is often hundreds of times smaller**, and the speed is greater than other methods, which are in further from the Shannon's bound than the realm for MTD .

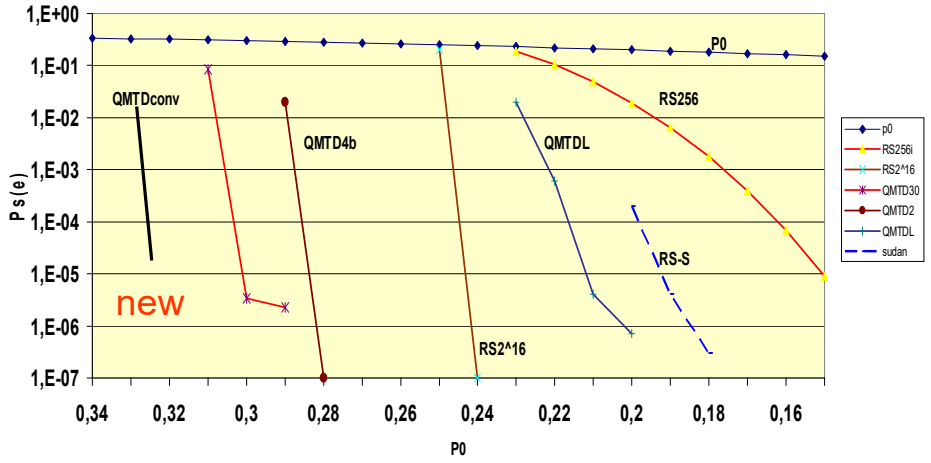
By the way, it's very funny that articles are still being printed in which **using 30 ÷ 50 and more complex expressions**, and even up to 20 links of other authors' there are **formulas** for various algorithms derived on probabilities of **erasure restore error**. **But it is impossible to check them!** And it always appears that "those" methods work when the probability of erasure in the channel is in 2 ÷ 5 times smaller than for MTD, which are presented on the slide. And they forget that their decoders are much more complicated than the MTD, and the quality of characters restoration is in very many times weaker. Recall again: **any analytical methods** for assessing resulting error probabilities near the channel capacity **are inoperative**. They can get these estimates only **at the models**!

We note after this that **for MTD** the probability of non-recovery, as for **OD,** is **always estimated as** $P_{ers}(s)=p_s^d$, where ***d***, of course, minimum code distance. **Is it simple?** Yes, it is! **That's all!**

Science fiction is getting 40 years old soon!

What are we talking about? It is about Reed-Solomon (RS) codes. But this is only for starting. Of course, we are again talking **about MTD**. But - about the very special ones!

Performance QMTD and codes RS



26

After the real full **triumph of the MTD** for BSC in 1978, i.e. our first binary decoder (it's a pity that it was not noticed at all then!), we really wanted to invent something instead of RS codes. They are applied everywhere. So, they are very necessary. **But they are short and therefore ineffective.** They were invented 60 years ago and after that theorists had no noticeable progress in non-binary codes. There are a lot of such codes, truly, but almost all of them are similar to RS codes. And they are used because **a lot of data types are byte** (digital music, photos, etc.), and they should necessary be protected, stored, restored and transmitted, of course. However, again, **short code is absolutely always a weak code**.

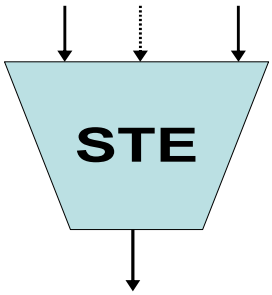
But long byte RS codes **do not exist at all**. What should we do?

The situation has long been difficult in this area of coding theory, also because **for non-binary codes it is impossible to create VA**. This is not surprising. General assessment of VA complexity for arbitrary alphabet q (when, for example, for a byte code RS of length n=q-1=255 characters and q=256) is equal to $\mathbf{N=q^K}$, which already for **K**>7 gives the value $\mathbf{N> 10^{15}}$. Who would do such a decoder, if its characteristics are too weak, because **K** is insignificant here?! (See above slide with graphs for RS and QMTD).

The last circumstance has especially strongly mobilized us to search non-binary MTD schemes. But after our binary MTD discovery, the search non-binary decoder has been lasting for over 8 years. And we have managed to invent it! We had got 4 patents for it. And thereby the **creation of the full scientific base for OT was completed!**

The mystery of symbolic MTD

Non-binary symbolic threshold element



Indeed, MTD for special non-binary codes that we are going to call **symbolic** in order to distinguish them clearly from the other codes, completed the last stages of creating a **new "quantum mechanics"** in applied information theory, we mean OT. Generally, now MTD decoding methods have been developed already for all channels, considered previously in the classical coding theory and our scientific school.

What has done? Those 8 years that we were looking for symbolic MTD were spend on finding suitable non-binary codes and working rules for the symbolic MTD decoders created for them. It had appeared that the structure of necessary codes should be similar to binary ones. And here is a **Symbolic Threshold Element (STE),** presented on the slide, had to be special and look for a decision about the place of the error as well as about its value in the range 1 ÷ (q-1), which, in principle, is not required in TE for binary MTD.

Such a rule of symbolic MTD was found! We patented STE as 4 ways of implementation, including linear from d complexity. True, the "theorists" immediately told us, that this is an empty job, because non-binary codes are very similar to binary ones. Nevertheless, **it was the third Nobel phenomenal result** (See page above).

Let us remind again, that among non-binary codes appeared very ineffective RS codes, created 60 years ago. A good non-binary VA like as we just found out, would never be possible to create. **That is all!**

As a result, **the symbolic "correct" MTD**, as a binary one, **has linear complexity relatively the code length *n* and converges to OD also!** Although there was absolute lack of results for non-binary codes over 60 years of the coding theory development, nowadays the problem of decoding non-binary codes turned out to be **"suddenly (!!!)"** completely resolved in OT. The symbolic MTD was completely described still in 1984. "Theorists" over these 35 years, however, **"have not seen it".** Then it is pity for them only.

It was real final of the "classic".

Now only OT is leading everywhere.

Almost everyone has appreciated the OT!

Our medals

The complete theory of symbolic codes, that completed the creation of updated Universe in coding theory, has become one of the most important reasons for awarding our scientific school **the European Union Gold Medal** **"For exceptional achievements"**, which only active scientists are awarded for truly outstanding research. At the first slides of our booklet readers could already make sure that the scientific and technological world community has long been followed our work. This **Gold medal** is visible confirmation of high evaluation of our results by **foreign high scientific institutions.**

The Prize of the Government of the Russian Federation is the second our scientific school medal.

But what about our, so called, colleagues? A few tries to publish our, of course, the highest achievements in the coding theory about symbolic codes in a respected magazine these "theorists" rejected, saying that *in our articles there was nothing new at all.* But it is the case to recall in this regard, we have received **4 patents for the symbolic codes!**

Our subsequent attempts to tell "colleagues" what we made such an extremely valuable, at the seminar of their "theorists" ended always very strangely. For example, they asked to tell about the foreign newest results. But they were not interested in our methods with linear complexity and optimal decoding at all. Especially bitter sorrow we felt at the last seminar about symbolic codes, where "theorists" ***once again (!)*** failed to deny **our Main Theorem**, they told us that we are undoubtedly setting out some foreign article, that we didn't understand ourselves, but we don't want to admit it. Finally, the "old theory" ended at all. Everything is clear here! **Now there is OT only!**

One of the most important awards of the OT School for inventions

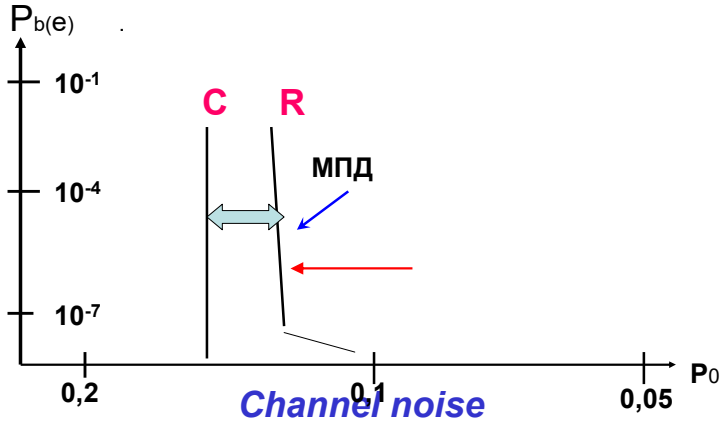
The slide shows **the Gold Medal** and Diploma of International inventions salon for our patent for the MTD decoder a high-speed version, which can be used in almost all organization of digital stream decoding processes options. Shown further on **p.28** the FPGA decoder also used this method, that **emphasizes its technological value.**

Let us repeat again:

coding theory is not a mathematical problem at all.

It is alloy of optimization computer technologies and theory aimed to solve the problems of increasing the reliability of digital data based on OT at the highest possible noise level and with minimal complexity. And the next slide:

On tending MTD algorithms to Shannon's bound



You can see here, that MTD methods work more and more close to Shannon's bound. But **this bound is absolutely elastic**. So nobody can work at R=C.

Where can you read this “in a scientific way”?

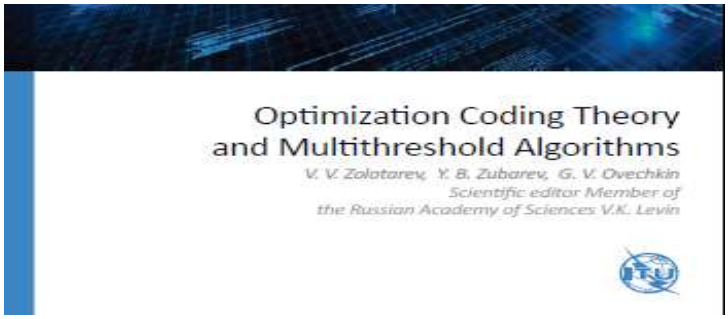
OT General Books

Главные книги по ОТ

Although on the previous pages we have already noted the qualitatively basic OT achievements, we have not discussed the access to serious scientific descriptions yet. It is very simple.

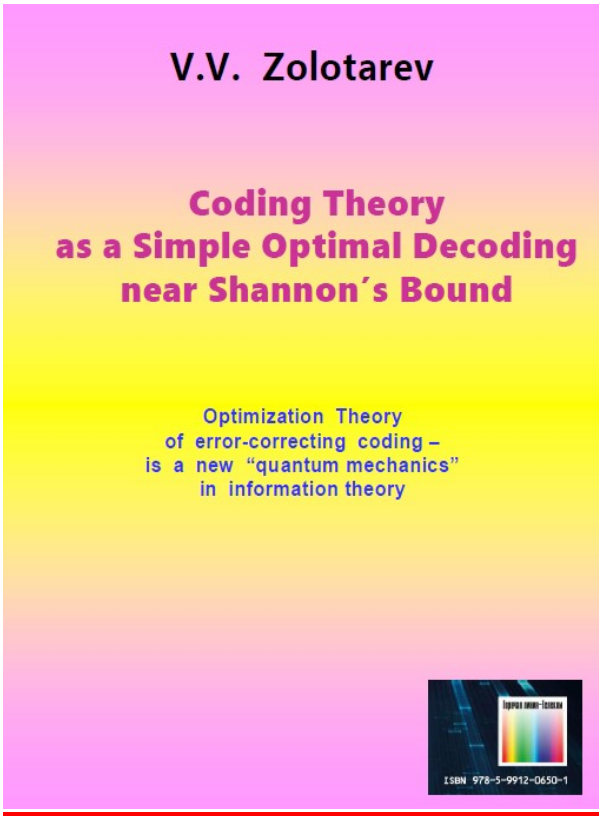
On sale or on our web-portals you can always find all our books in Russian, including recently published ones. Some of them are on the slide above. Please, open **www.mtdbest.ru** , page “Our books”, there you can find information about all the main monographs, published by our scientific school.

Our large book published first by ITU



This collective monograph was published in 2015, the year of 150th Anniversary ITU, on the recommendation of its Secretary-General after a thorough examination. This book outlined all the main results of OT, including **total theory of error propagation (EP)**, which opened up wide avenues for applying OT methods and MTD algorithms in the vicinity of Shannon’s bound, i.e. when $R \lesssim C$.

It seems to us very important now to submit here our second complete editions monograph **in English** about OT and decoding near Shannon's bound, which is posted on our portal **www.mtdbest.ru** also, like the shown above book and several others. You are welcome to download it without any restrictions.



We believe that special role in bringing to the scientific and technical foreign community the latest OT results belong just to this monograph "**Coding Theory as a Simple Optimal Decoding near Shannon's Bound"** published in **2018**. It is large scaled and widely interacts with our web-portal in a variety of ways, so that its real informational content is many times more large than its physical size. Of course, the enormous usefulness of learning all previous OT monographs is also undeniable. **Our reference book about coding theory** in Russian is very popular also.

Are you ready to help publish the second modern reference book on coding methods?

It is especially important to point out that another innovation has been prepared in our new book. As we have already wrote, in no small measure the huge advantage of OT before the rest of the science about coding, due to the fact that the development of OT during all 50 years of creation and studying MTD was based **on the accelerated highly intellectual software formation**. All scientific groups who want to conduct researches in the field of coding theory must create it literally from scratch or find other ways of development for yourself.

This is an extremely important and very difficult task.

In addition this book unlike the previous ones, allows interact now not just with some optional software, demo software, but offers to turn to a **large number of special software platforms** at our portals, that are full-fledged software resources. Access to them allows you to get simple instructions and immediately to start research at these platforms your various types of codes and channels with a wide variation of working methods with MTD, including the analysis of those codes that you wants to choose and estimate yourself.

Concluding the presentation of the most essential foundations of the new coding theory, our OT, we want to draw your attention to the fact, that there are extremely simple organization of the decoding process of absolutely all codes based on, of course, slightly different versions of MTD, which, however, are built on very general unified principles, led to another extremely important result that has already been briefly mentioned. And **this result is the tremendous speed of all MTD decoders.** In program form, for example, at the C++ language, MTD demonstrates for a high noise level, decoding speed ~ 10 Mbit / s in Gaussian channel at d ~ 11 and about 10 iterations (attempts to decoding) at the laptop with a clock frequency of ~ 2 GHz.

Comparison with a small number of other types decoders for which data on software performance exist (alas, after 70 years of coding theory development, similar data are extremely small) shows the advantage of MTD in speed up to 50 ÷ 1000 and more times! And the decoding speed in the most code applications will always remain as an important advantage as noise level at which the decoder is operating, or total data reliability after error correction. **Check this, please**!

Now we show our monograph in Russian, which had been published before newest English book about Shannon's bound.

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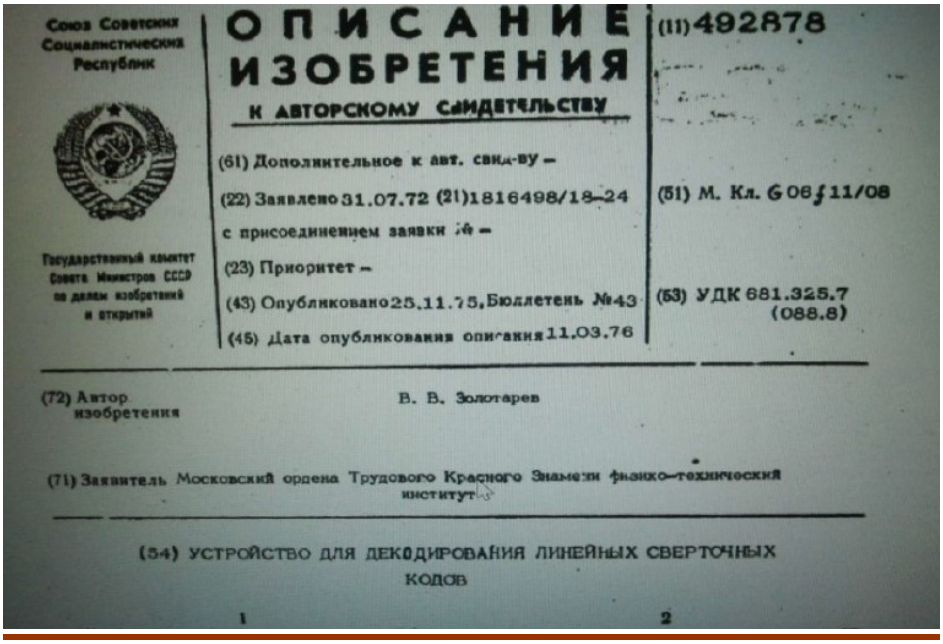
Below you can see the author's certificate (patent) for the MTD invention with priority at 1972 in SU, that our scientific school have got still in 1974. We successfully work in the field of advanced decoding algorithms for a long time.

First soviet patent of optimization decoding



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Государственный комитет Совета Министров СССР по делам изобретений и открытий

ОПИСАНИЕ ИЗОБРЕТЕНИЯ

К АВТОРСКОМУ СВИДЕТЕЛЬСТВУ

(11) 492878

(61) Дополнительное к авт. свид-ву –

(22) Заявлено 31.07.72 (21) 1816498/18-24

с присоединением заявки № –

(23) Приоритет –

(43) Опубликовано 25.11.75, Бюллетень №43

(45) Дата опубликования описания 11.03.76

(51) М. Кл. G 06 F 11/08

(53) УДК 681.325.7 (088.8)

(72) Автор изобретения В. В. Золотарев

(71) Заявитель Московский ордена Трудового Красного Знамени физико-технический институт

(54) УСТРОЙСТВО ДЛЯ ДЕКОДИРОВАНИЯ ЛИНЕЙНЫХ СВЕРТОЧНЫХ КОДОВ

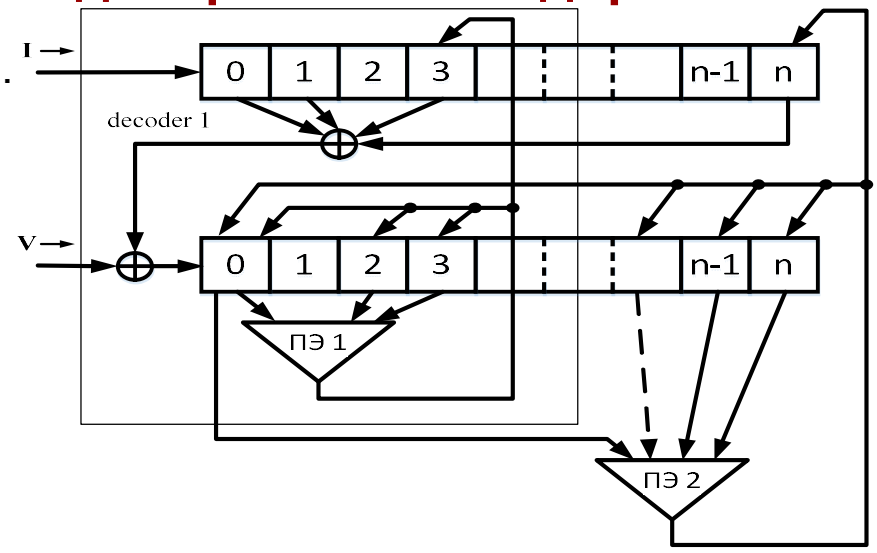
1 2

Here you may see some of our patents, which total number is ~40

Special paradigms of Optimization Theory

Principles of divergent coding

Принципы дивергентного кодирования



Solving improving coding efficiency problems based on MTD decoders and various AV modifications, aimed at achieving OD solutions for the codes used near the Shannon boundary, i.e., directly near the capacity of the communication channels, has led to the special research and development methods creation, most of them has been patented. One of the most important paradigms formulated in OT, became **the divergence principle**. It is a non-concatenated increase the code distance of the code construction. Usage divergence ideas example is given on the slide where the first threshold element (TE) of the convolutional code decoder has a number of inputs, which it makes decisions based on. But the second TE makes decisions using by 1 number more of inputs. This TE further reduces the number of remaining errors, helping the first TE to correct its flow channel errors better .

In fact, the **process of two TE interaction is very complicated**. However, with the correct ratio of the code parameters and both TEs, it can be a small effect of the decoding result improving after second TE. Of course, the proposed scheme can be considered as "first TE" in another external divergent scheme, etc. It is essential that even such a simple scheme adapts well to working conditions, using **software tools for optimizing parameters** encoder and code. MTD development technologies based on divergence ideas are extremely useful, and **the algorithms themselves are very efficient also.**

Direct metric control

Principle of direct metric control

- • VA - in convolutional or block version - the only method that accurately measures the distance between the received vector and all potential decisions.

- • MTD is the only method that measures the distance between just one vector that is intermediate decision, and received vector.

- • **There are no other similar methods!**

This slide is very toughly and unambiguously shows that now there are only **two classes of algorithms** that accurately measure **remoteness of their decisions from the received vector.** VA at the same time finds among exponentially growing with increasing *n* set of possible decisions the nearest vector to the accepted vector. This is OD, since it is most probable. But, for example, **VA** has to remember always all these decisions.

MTD does not store all valid decisions. It contains the current best unique decision for the moment. It should find the only required (also from an exponentially large set) OD decision. **MTD does it with linear relatively *n* complexity** even **near Shannon's bound** if a code with a minimum level of EP is used.

Why **VA** in all modifications and all types of **MTD** in OT allocated to a special class of **decoders** with **direct metric control (DDMC)**?

This is almost immediately clear. **VA** paid by enormous complexity for the fact that it does not lose the OD decision in the process of work and then transfers it to receiver. **MTD** does the same, but pays for the possibility of achieving an OD decision possibility by the following:

A) MTD should take only a code with a small EP;

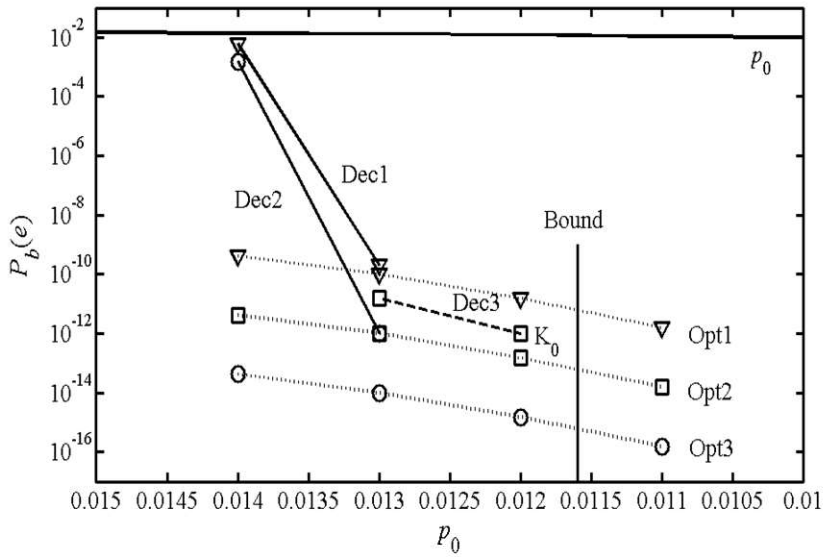
B) It leads necessarily to large delays in the decision;

C) There is always a small area between Shannon's bound and the MTD workspace, which, however, is gradually decreases due to the OT school's efforts (see slide at p.18). We emphasize that this is a very a **small fee for the linear complexity of the MTD instead of exponential computation volumes.**

But for all other methods it is much more difficult than for **VA** and **MTD** to achieve OD decisions, as they simply "do not see" their goal - OD, because they do not measure these key distances. Even much more great complexity of such methods does not solve the OD achievement problem.

It is confirmed historically!

Decoder for flesh memory



The set of OT paradigms made it possible to solve an important rather self-contained flash memory problem. This is the name for the class of errors correction tasks in unreliable memory of a limited size, when they especially need highly reliable decoder result. It is because otherwise all data and software modules, stored in it, could not be used for anything. We emphasize that there are practically **no other decoder systems with particularly significant results.**

It is also very important that the decoding problem statement in this the case changes significantly. Capacity requirements for these channels are excluded, and the decoding speed becomes especially significant. Well, of course, the distortion of stored data occurs randomly and independently in bits or bytes. The probability of MTD error, for example, per bit $P_b(e)$ **should be** $\sim 10^{-15}$ or even less, which is the main point in these tasks.

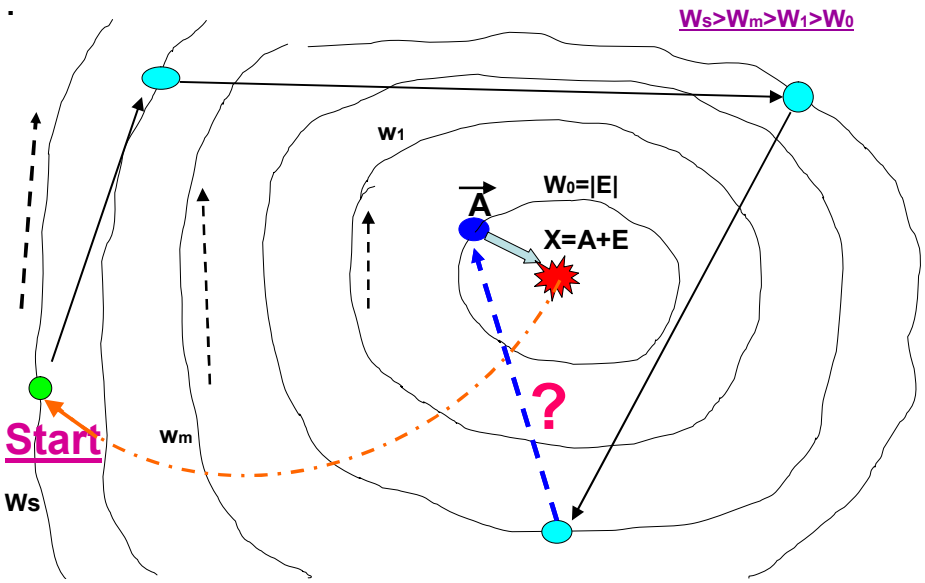
Result for a simple decoder with a bit model of distortion presented on the slide for a little redundant code in the simplest version Implementation of MTD decoding and parallel concatenation. When additional cascade based on parity check codes would be used, then they can get the result even easier. With a byte model of memory distortion, the circuit decoding scheme remains the same.

After reaching $P_b(e) \sim 10^{-12}$, the implementation of levels up to $P_b(e) \sim 10^{-18}$ **does not represent any difficulties for MTD,** although confirmation of the achieved reliability will require some efforts.

That's all about the OT basics!

How else can we look at the MTD work?

One of our students looked at the MTD scheme on page 8 in our comics about OT and has drown the next picture in a minute. When we asked him, what is it, he explained us that this is how he sees the MTD process.



Since the code under discussion is linear, let's assume that there is a high probability of distortion in the transmission of code characters and a null message **A** is sent. The choice of the transmitted message, which is a code word, does not affect the errors probability in the decoder decisions. During transmission the noise vector **E** was added to the code vector **A**, whose resulting weight is equal to |E|. In our case, the weight is the number of units in the binary vector. The decoder will receive the vector **X=A+E**. OD must find such a code word that it would be the closest to the vector X among all possible ones. Since **A** is a zero vector, the weight of **X** is the same as that of **E** : |X|=|E|. And let the message **A** be the closest to **X**, i.e. the code word **A** is the decision of OD.

We immediately asked the student how he would tell us about his image if noise vector **E** has such a large weight, that **A** is not the decision for OD. He almost immediately pleased us with a clear answer that in this case absolutely nothing changes at all, since he talks about the process of convergence of MTD decisions to the OD decision, but not to the correct decision. So in order not to interfere with the understanding of the image, our student has immediately suggested that the vector **E** is not so "heavy" and the sent message **A** is exactly the OD decision.

And then it turned out to be even more interesting. As we know, in MTD, as in many other decoders, the simplest procedure for calculating the syndrome vector for the received message is performed first (see the books about it).

And then we get an initial version of the MTD decision, which the process of finding the OD decision begins step by step from. This start position is shown on the blue "**Start**" dot at the edge of the slide, additionally marked with the $\mathbf{W_s}$ symbol. This sign indicates the weight of the difference between the start point and the received **X** vector. This point lies on a certain circular line, there are all decisions, that have the weight of the difference with **X** equal to $\mathbf{W_s}$. The weight $\mathbf{W_s}$ is always really much more than |E|, $\mathbf{W_s >> |E|}$. On the inner circles lie other decisions, the distances Wm to which from the vector **X** decrease as they approach to **X**. The total number of decoder decisions lying on all different circles increases exponentially with the code length n.

In this case, the decoding process becomes such as, according to Fig.**XXI** of the block MTD, all data in it move synchronously and cyclically. The threshold element gradually changes and improves the controlled information symbols. And in the upper diagram during error correction the decoder looks at the current $\mathbf{W_m}$ weight circle (but first, of course, on the "Start" $\mathbf{W_s}$ weight circle!) for the ability to move from it to one of the inner circles, when each of it is a geometric place of decisions, that are at some equal distance $\mathbf{W_n}$ from **X**, and $\mathbf{Wm > Wn}$.

However, the threshold element in the MTD is such, that the transition to the inner circle is possible only when the new decision at this inner circle of lower weight differs from the previous decision in exactly one information symbol. And this condition is the greatest problem of majoritarian (threshold) algorithms, which was formulated and then successfully completely solved by the theory of errors propagation (**EP**). But if **EP** problem is not solved, then usually the decoder will run around a circle of some weight $\mathbf{W_n}$, $\mathbf{W_n > W_0}$, after some time. The possibility of such a failure is shown by the last dotted arrow to the decision **A**, which is emphasized by the red question mark. There are no any guarantee that the algorithm will achieve the final correct decision **A** at the end of the decoding process. But if you use all the features of the EP theory and build only the correct codes according to the **EP** criteria, then even with a large noise the difficult search on the circles of the student's diagram would almost always continue until the decoder gets to the decision **A**, which is the closest to the received vector **X**, as we have agreed before.

Our student also said that the current condition for the uniqueness of the changed symbol in the next MTD decision after each message correction event is very strict, but it is **not necessary at all**. He has suggested searching for the other crucial rules for MTD **in order to change many symbols** or even in some other way, which can improve the decoding characteristics at high noise levels.

It is quite obvious that the main condition here – preserving the linear complexity relative to the code length ***n***, - can be fulfilled.

But the search a specific type of new decision function is a very difficult task. It may soon be solved by our student or other participants of the OT scientific school. It will be the next step in the further development of the error propagation (**EP**) theory for MTD. I wonder who will solve it. Yes, who will do it? **Our student, we, or maybe YOU**?

Decoders of this kind are absolute champions forever

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The multithreshold decoder **(MTD)** for satellite and Space channels, raises efficiency of their usage in 3-10 times, including **channels up to 2Gb/s**

MTD for the Space, optical channels and flash memory

On this slide hardware decoder created in SRI RAS is shown that implements the multithreshold algorithm (MTD) at the **FPGA ALTERA**, created many years ago. It uses several patented ways to improve the performance of decoders. These methods already confirmed their effectiveness on other MTD decoders, in particularly at the **FPGA Xilinx** .

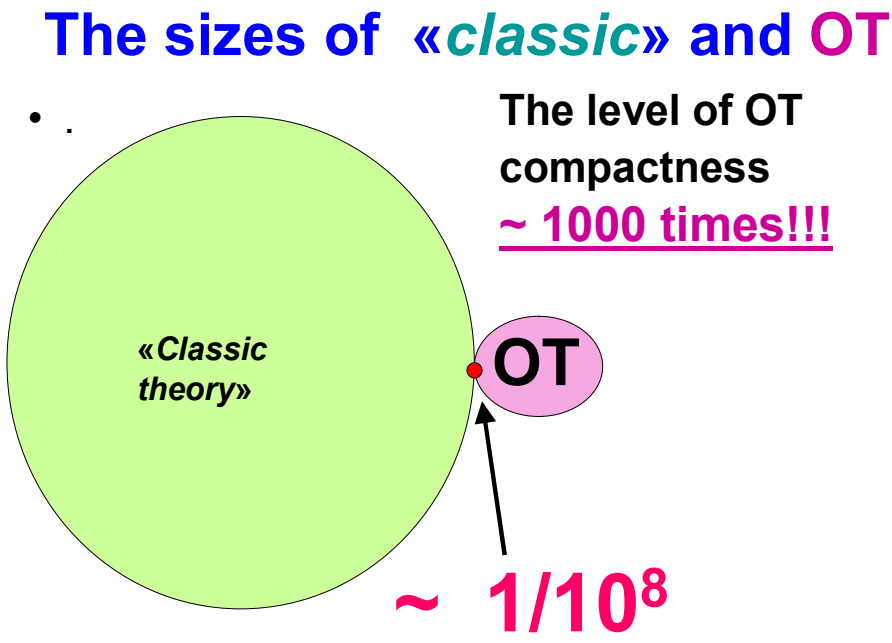
This MTD demonstrates the successfull work in the Gaussian channel with a high noise level and provides speed decoding of input convolutional code stream **over 1 Gbit /s** information symbols at code rate R = 1/2. It corresponds **theoretically to the maximum possible hardware decoding speed**, when the speed of data movement through the registers of the decoder is the same as if they move along the usual "empty" shift registers of a given element base at the maximum allowable clock frequency for them.

The threshold element implemented in MTD at FPGA is organized by one of **the four school OT patented methods** and demonstrates planned the level of its speed, which allowed it achieve the highest hardware decoding speed, inaccessible to other algorithms.

It is also important that **the simulation of several of our codes** on these and other similar **FPGA** once again confirmed the achievement of all planned performance characteristics previously obtained at the basis of theoretical and software development and research.

About OT perfection and its compactness.

A very important question is the place of OT and its relation to the former "classic" coding theory. A very conventional high-quality picture illustrating these relationships is the slide below, which nevertheless provides clear answers this question.



An analysis of all OT monographs published by our scientific school in this Millennium shows, that they are all systemic-philosophical treatises on information theory with extremely limited usage of complex mathematical methods. Some exceptions are those sections of OT that relate to the basics of error propagation (**EP**) theory and its applications to specific codes. It is quite clear that such a new important and very original section of coding theory, which no scientific groups in Russia and abroad have been able to discover, research and understand over the past 40 years, could not be very simple.

Moreover, after a complete analysis and such a complex phenomenon as **EP** description, the conclusions and technologies followed from its understanding were also clear and natural. **EP** has shown that it is necessary to construct codes with the lowest possible dependency of checks relative to decoded characters among themselves. This conclusion, which could not be made without **EP** theory, led to simple (of course, relatively simple!) algorithms for creating such codes, which also fit into the concept of simple methods with clear and understandable final criteria for the quality of codes and decoders.

The total number of new mathematical relations in the characteristics of OT methods turned out to be quite small and is now limited to just a few dozens new simple formulas. And from the previous applied coding theory, only a few simple transformations are used in OT, which allow us to calculate the

probability of error in the first character of the used binary block or convolutional code $P_1(e)$ for a binary symmetric channel {BSC). All this, however, allow us to calculate all the necessary characteristics of decoding by OT methods in all classical channels considered in the coding theory for any algorithms of our new "quantum mechanics" in the information theory.

The described situation is presented on the slide, which shows the almost completely independent former "classical" and new optimization part of the applied coding theory. A small red spot connecting them shows, that several expressions make up the proposed J. Massey method for calculation the probability $P_1(e)$. This single point unites them. OT- is a completely rewritten applied coding theory. The ratios shown on the slide for the dimensions of these two theories corresponds roughly to three decimal orders of the informational volume. The conditional share of formulas for $P_1(e)$ is really insignificant and the specified part of them is actually only their qualitative role. Well, finally we note, that the entire OT is a very clear and completely understandable theory. **So learn it, please!**

OT is very simple! But the results are the best and optimal!

-=-===-==-==--==-

Had OT really solved in general the Great Shannon's problem, stated 70 years ago? Positive answer this question was formed in all our monographs of this millennium. Indeed in these books (and in [4,7] also!) many results were submitted and had shown that for block and convolutional, binary and non-binary (symbolic), low-, middle- and high rate, erroneous and erasure, basic and concatenated (and others!) codes we can construct now MTD and BVA decoders with extremely high characteristics near Shannon's bound. We had checked and analyzed ~ 100 such a very different and successful clusters.

Our scaled work here clearly proved the fact: - Shannon's problem is entirely solved! You and we may turn to solve the next more difficult problems of information theory.

Well, Good Luck to you and us!

And now - about the general situation in coding

See some opinions and facts related to OT theory. They are very simple and clear. Therefore our further comments will be shorter.

So, we look at the following slides:

The reasons for the "classics sunset"

- The problem of the correlation of the theory possibilities and experiment has particularly escalated in the 80s due to the rapid development of computing technicians. Some publications on this subject were found even at RAS website. We cite from such an article only some phrases:
  - "**....** Hard confrontation between computer simulation and theory based on mathematical methods – **it is Disease of our Century.**
  - **...** A theorist working in any scientific field knows that not all problems can be solved analytically: for the vast variety of problems, it is not possible to get exact or **even approximate solutions**.

... Combining the capabilities of theory and modeling is **a special profession!!!** It requires combinations of skills and talents necessary **for theorists** of the last century and specialists **in computer modeling** of today."

* 09.09.2009 - Nezavisimaya Gazeta, ("Independent newspaper", "**IN**"), Moscow, "The number raised to the absolute".
Yu. Magarshak - professor, executive Vice President of the International Committee of intellectual cooperation

- { **RAS** website or https://mtdbest.ru/articles/theory_modell.pdf }

It became an accurate and strict diagnosis of the "classical" coding theory. It left the scientific scene because there were no any single tested real decoding algorithm from the "classics" **during more than 30 years at all**!

And conclusions:

- Theory and experiment must mutually accelerate, control and support each other.
- As far as theorists "about codes", they are in deep conflict with reality.

What does it means?

All applied decoding problems have united quality criterion:

"noiseproofness - complexity - veracity".

BUT! Nobody can ever count them!!!

And why? The coding theory is not a mathematical problem at all!

Are there any foundations for such a statement? During 60 years we were said that the coding theory is a great and a very interesting mathematical problem. But just this is a very strange situation.

Let's see it more concrete.

Now it is impossible to calculate **distance between Shannon's bound and work domain of any decoder**. But this is a clear measure of noiseproof abilities.

Theoretical estimates for decoder complexity even for the same algorithms appear very different. These results do not coincide, if researchers use various methods. And it is difficult to achieve agreement upon similar results. Moreover, near channel capacity similar results are essentially different.

At last error probability for all decoders may be calculated only for very low noise level. This is not interesting. But experience of last 50 years have shown, that **for noise level near Shannon's bound it's impossible to calculate error probability for effective algorithms**.

So, "classic" theory has no any useful results near Shannon's bound at all.

Let's repeat:

It just means that coding theory is not mathematical problem!

But we know that OT has already solved grand problem of Shannon on the base of solid union of delicate theory and multi-purpose scaled intellectual **software, that had been developed during 50 years**.

Since 1971 scientific school OT applies research technology, unifying theoretical (analytical!) methods with total experimental researches, in particular, based on **software platforms, which allowed to set and solve problems**, **even to formulate which only in terms of theoretical foundations is generally impossible!!!**

It seems, compulsory software development, except OT school, nobody has started to work on. Nobody writes anything meaningful about the complexity of new decoders. No one could ever be able to calculate **the main parameter** - **probability of decoding error near the Shannon's bound.** It seems that no one needs it, is not it? Nobody is able. **BUT!!!** OT methods can do it in 5 ÷ 7 minutes. **After all, there is no other way to do it!**

However, the slogan of the end of that Millennium: "Programming is the second literacy" has now become completely irrelevant and, in fact, completely lost.

And here is the balance of results on coding theory in the world and in the Russian Federation

History of major discoveries in coding theory

- **World Progress**

Hamming Codes	- 1950
Codes BCH and RS	- 1960
Thresold decoders	- 1963
Concatenated codes	-1966
Viterbi algorithm	-1967
Turbo codes	- 1993
LDPC codes	- 1961-98
Polar codes	- 2009

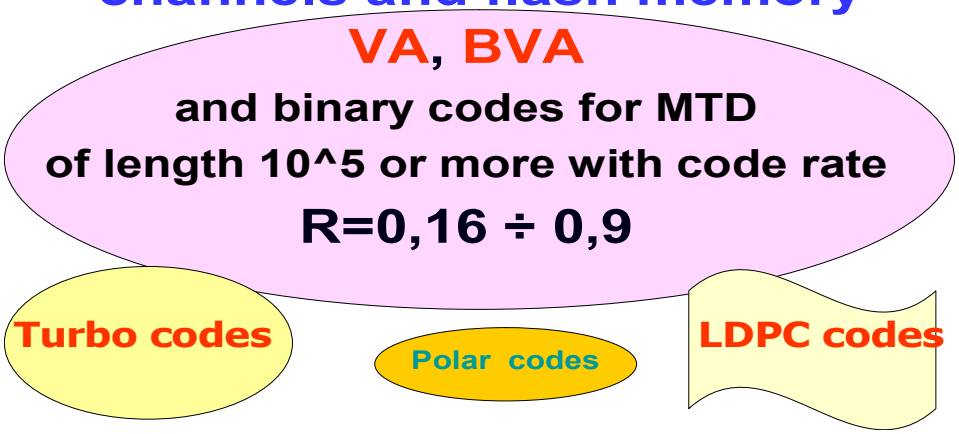
Achievements of USSR and RF

MTD	- 1972
Codes with low EP	- 1975
GCC	- 1975
Symbolic codes	- 1982
Symbolic MTD	- 1984
Parallel codes	- 1985
«Instant» codes	- 2007
Cocatenated MTD	- 2009
Optimization Theory	- 2010

You can compare all these applied achievements among themselves. It should be noted that among such achievements of the Russian Federation, the school of IITP RAS revealed GCC codes, and all other results belong to the scientific school OT. Russian achievements that are marked with red – are inventions.

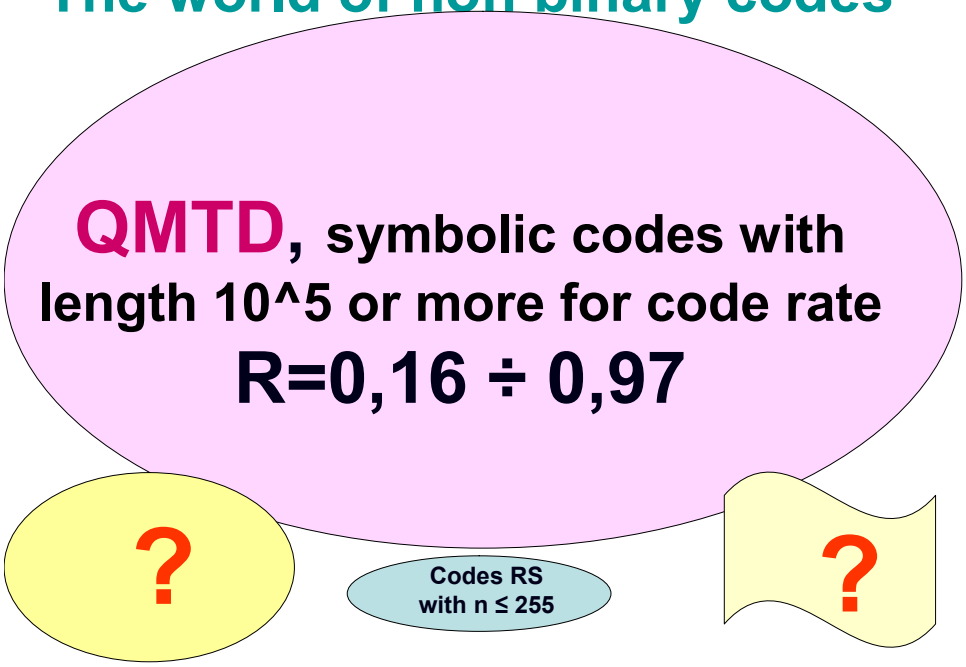
Russian results are wider much more. The divergence and DDMC are our original results as well. **And it is very important!**

The world of binary codes for satellite as well as optical channels and flash memory



Here we list those methods that OT circumvented by strict complex criteria " **noiseproofness-complexity-veracity**". Look for all the details of such a comparison in our reviews on decoding algorithms. But real numbers about complexity for the "polars" and other "achievements" cannot be found anywhere, because no one published them. **They are all extremely complicated**!

The world of non binary codes



This slide shows the situation with **non-binary codes**. We have even nothing to compare ourselves with in the whole area of non-binary codes, i.e. here **absolute leadership has been achieved for OT and symbolic MTD** too. And non-binary "polars" are the complete failure: their complexity is **up to ~ n^3** and even much more. **Where is more?**

And some opinions:

- Zolotarev algorithms with linear complexity strive to the optimal decision .
- • It is significant that a number of scientific results author's school are discoveries in theory coding, which is absolutely true points out the scientific editor of this unusual book Academician of the Russian Academy of Sciences N.A. Kuznetsov.
- • Analogues .to Zolotarev's codes in simplicity implementation and noise immunity does not exist.
- • OT has become a “quantum mechanics" of information theory based on global functionals extremum searching in special discrete spaces.
- •
- Professor of Higher Mathematics Ryazan state university of Radio Engineering, director of the System Analysis Laboratory, doctor of physical and mathematical sciences Mironov V.V.

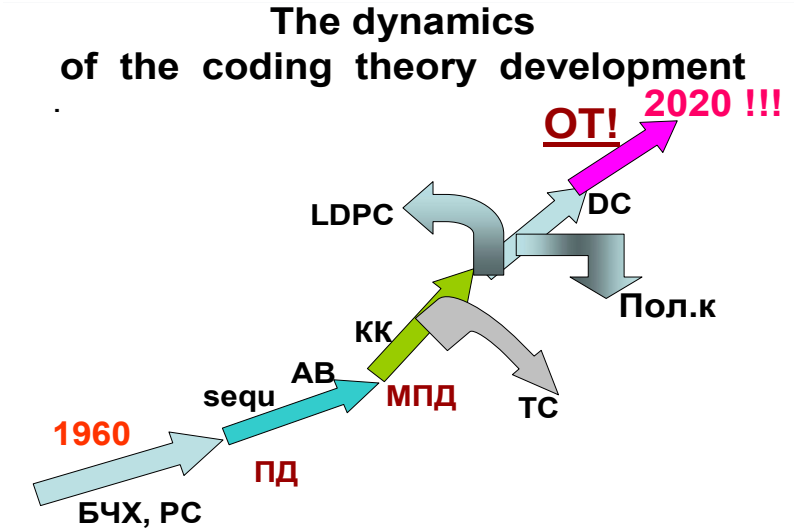
RAS Opinions

RAS on Optimization Theory

- **..... Five generations of MTD created at NIIR ....**
- **• ……. Zolotarev's outstanding achievement has excellent scale analogy in physics beginning 20 century. She was in crisis and could not solve many new tasks. Quantum Theory opened by Small Group future Nobel laureates for physics absolutely unusual new horizons.**
- **• Similar absolutely grandiose work committed and Zolotarev's scientific school in theory information. Now any levels are possible data reliability and processing speed at very high channel noise. This main applied the problem of coding theory is completely solved simplest means and with an absolute minimum resources.**
- **• The triumph of Zolotarev's theory around the world network portals of his scientific school also confirm ~ 100’000 people visit every year. ……**
- **• (Signed: 03/10/2015). Corresponding Member Yu.B. Zubarev**

- …. the work of the Russian scientific school on coding theories have overcome long-term crisis theory and based scientific and ideological revolution, replaced a significant part of the main coding theory paradigms created conditions for its development on a completely new basics. It can be said that in theory coding has formed its own "quantum mechanics". She is difficult, but fruitful. There are no other ways.
  • Corresponding Member of RAS Yu.B.Zubarev
- • Magazine “High technologies” 2016 year

We restrict ourselves further only to add, that each monograph of the scientific OT school were edited by academicians of the RAS for the last 15 years.



And this slide briefly shows the difficult path of coding theory from 1960, when the algebraic stage of the theory began. And then there were thresholds and sequential methods, concatenation, VA, then **MTD, DDMC** and finally **OT**. But turbo, LDPC, and, of course, polar codes too distracted the scientific community from the goal - simple decoding near Shannon's bound. In 2020 year and further only OT and its methods will go successfully. **Yes!** They must always choose scientific technologies the right one!

But why so? Because **OT provides**: 1) - minimal complexity, 2) - optimal decision and 3) - extreme proximity to the Shannon's bound! **It’s absolute result.**

Application of the most powerful coding systems for sources and channels

• 1. **Channel coding.** Increases data transmitting veracity at 2 ÷ 9 decimal orders CG ~ 8 ÷ 15 dB.

• 2. **Source coding**. Data compression in 2÷5 or more times is achieved.

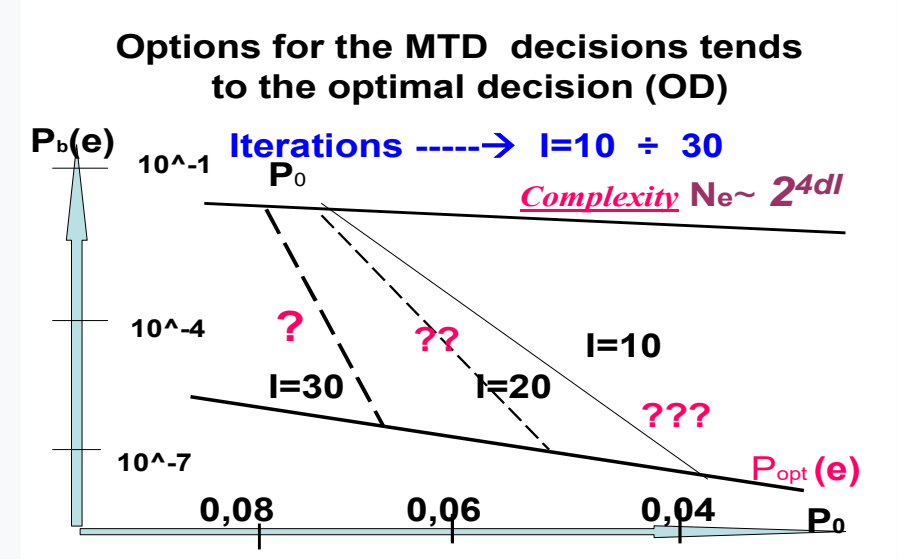
• 3. The total final energy gain from coding theory methods applying is up **to 40 ÷ 100 times**!

And this slide is about the overall code gain from applying all coding theories: both the source and the channel.

Up to 100 times!

But it is so if we do everything right.

This is how great the role of science in the digital world!



Here is the last informative slide to complete the extremely simple OT methods description. On the graph with realistic numbers for the channel with independent errors and error probabilities $P_b(e)$ of the MTD algorithm different colors indicate characteristics of MTD with a different decoding iterations number.

We would like to remind: the approximate curves $P_{opt}(e)$ for different numbers of decoding iterations appear in a computer experiment in 3 ÷ 5 minutes, sometimes in an hour. The approximate complexity of the analytical estimates of $P_{opt}(e)$ should be much larger as shown by the exponential estimate of N_e on the slide i.e., it can be obtained never. In general, all decoding methods are just like that: parameters of reliability in the domain **close to Shannon's bounds can not be calculated at all.** And OT methods evaluate resulting probability $P_b(e)$ almost instantly: over 50 years of OT development all necessary software platforms were created, with a large stock. Many of them are on our websites and we distribute them freely. All this increases our researches speed in hundreds of times. We invite everybody to work with us!

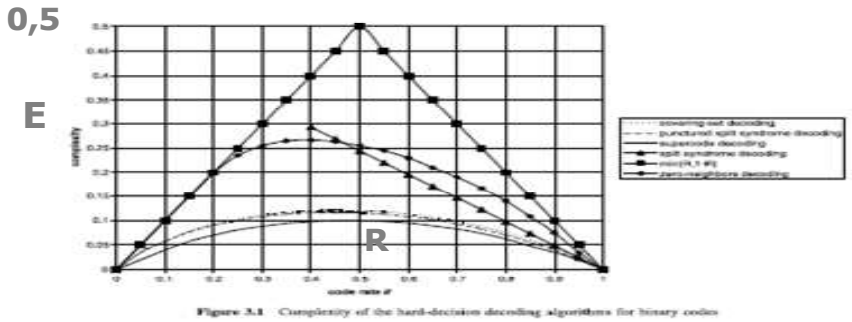
We are finishing. The picture shows the working technology in OT. For given parameters of the coding system, evaluating the probability of the usual TD, we find the required values of the code distance *d*, not forgetting to test potential parameters of **BVA** or **MTD** decoding with respect to computational rate of the channel and its capacity. If speech is not about new results near the Shannon's bound, then optimization methods, the number of which for each algorithm development cycle is not more than 5, will bring decoder veracity parameter to the bottom P_{opt} (e) curve on the picture. When using **VA** cases, these cycles are even shorter, since all **VA** are optimal methods. So OT is very simple. But only if you have a software! Join us now.

Only this is a science!

So here is the difficult and joyful finish: let us be leaders!

Last comments on the theory that came down from the scene

- **Complexity for block codes N~2^En**



For optional code when n~10000 →→ N~10 ^{200} operations!
It's much more then number of atoms in the Universe!

On this page we have prepared for you the criteria by which it will always be convenient for you to select scientific and educational literature on coding methods. We are going to show those some "ideas" that you can see and then freely to close such books or articles "about codes". **Forget about them!**

At the picture above we show a graph of the "most important achievements" of the coding theory completed recently in a large number of doctoral dissertations. They are "new improvements" in decoding complexity compared to exhaustive search. i.e. these "doctors of sciences" had reduced the exponent of decoding complexity by 2÷ 5 times. So there are $N > 10^{200}$ operations for standard code parameters with n~10000. By the way, it is much more then number of atoms in the Universe. **It's quite new, isn't it?** But since 1981 it is well known that in OT codes may be optimally decoded with linear complexity: when N~n. Do not read those books and articles!

2. A huge number of "doctoral" works tell us, that convolution-based block codes can be decoded with complexity $N \sim 2^{2K}$. And even students are very aggressively studying this way! Here K – is a convolutional code polynomial length. Don't read them either! These are empty theories. In OT the complexity of such decoders is $N_{OT} \sim 2^{K}$! Like it is for classic VA! And these block Viterbi decoders (BVA) **are patented !**

3. There are millions publications about how good Reed-Solomon (RS) codes are. Yes, it is! They were very useful in data storage and communication systems. But they are always really short and have a complexity $N \sim n^2$. Formally, the complexity of N is somewhat less, but it is not significant. That means that they are ineffective and slow. Symbolic (non-binary!) MTD with any lengths are decoded optimally with complexity only N ~ n. **Catch the difference!** This is also described in detail **since 1984 and patented**. **Use it together with us!**

4. A lot of unjustified oddities created methods for transmitting RS codes over binary channels. But these are poorly coordinated coding systems and modulation methods. Their effectiveness and usefulness are very small. **Forget about them, please!**

5. The games are boring with polar codes and with channels where the received symbols are unusually erased somehow. And no any data on their complexity at all! But it is (recall: when thousands of very complex decoding iterations are "there"!) - huge calculation volume. **Ignore it too, please!**

Well, let's select a single page to evaluate the achievements of those authors of articles on codes who cannot be attributed to the scientific school OT. It is impossible to consider all articles. But, nevertheless, let's to take a look at recent articles of an other plan that the scientific school of OT offers us.

Look at the slide where we have compared two methods: decoders LDPC and MTD:

Estimations of decoder complexity for BSC channel.

- LDPC: IITP, "ITP", **2009 year**
- 15 pages A4, tnr-10, > 50 formulas,
- 15 references to literature. Ne~0,0002n !
- Complexity calculated – **N~ n * log (n),**
- effectiveness ???
- - = - = - = - - = - = - = - = - = - = - =
- MTD: "ITP", **1986 year**,
- 2 pages, A5, tnr-12, 2 formulas.
- Patented. The decision is OD ! Ne~0,05n,
- Complexity – linear, N~n **since 1978.**
- + 7 monographs, results -> modeling!!!

Here for BSC theoretically (50 formulas and 15 links!) they had shown that LDPC decoders with R = 1/2 correct a fraction of errors of the order of 0.0002n, where n - code length . But even 60 years ago, BCH codes corrected a fraction ~0.03n, that is ~150 times more, i.e. better. And **MTD,** as it was shown **in the same journal**, corrected the share of more 0.05, i.e., ~ **250 times greater**, **in 1986**. And all it is because in the article about LDPC error probabilities are recently obtained purely analytically. Maybe, the algorithm was ineffective there? Modern publication. **And why is it so? What for?**

Let remember again: nobody can now and in future count the decoder error probabilities for any algorithms when the noise is large enough.

After 4 years, the authors of the above article improved and corrected the result. The output error rate was **increased in 10 times**. But this is still in ~25 times weaker than that of **MTD-86**.

Analytical methods for evaluating decoders in a Gaussian channel we had not found anywhere. But MTD models demonstrate experimental characteristics and in these channels near the Shannon's bound. **There is nothing to compare with**.

We have found analytical methods, that are different from our evaluations for erasures correction. But MTD decoders, as it turned out, successfully worked in the channel, where the erasures fraction was more than in 5 times higher than the permissible level for "those" codes that are also much more difficult to decode than MTD. In 5 times - it is too much. This is a lack of all pure analytical evaluations. They really can never be verified (see the link above at "IN") at all.

Finally, for non-binary codes, the advantage for symbolic codes in error probability in the channel was more than three times. Moreover, the complexities of "those" algorithms were at **2 decimal orders greater** than for MTD. **That is all!**

Thus, there **are no publications** about decoding methods in different journals of the last decades could even come closer by hard criteria **"noiseproofness-complexity-veracity"** to the level of the algorithms OT in the channels with random distortions considered in theory.

So start modeling! But change the methods!

Conclusions. Applied coding theory as a search for methods close in reliability to OD and with a linear of code length decoding complexity near the Shannon's bound for all classical channels considered in coding theory, fully completed on the OT basis. Classical coding theory passed the baton of leadership to OT, the **new "quantum mechanics"** of the information theory. Created innovative optimization OT technologies will be developed and applied when creating decoders for more complex classes of channels, types of signals and kinds of communication systems.

* * *

We give now list of the basic OT books. Above we already learned what types of publication we would recommend to refuse. Yes, perhaps, our opinion will be the reason for very sharp criticism. But a large clan of "scientists" had created the deepest 40-years crisis in our very important area of digital computer sciences. Nobody could decide to challenge to their opinion until recently. But we believe that after the complete solution by our scientific school the Shannon's problem and the transformation the previous complex infertile applied "theory" into the new **"quantum mechanics"** of information theory (how do we evaluate our OT), **we have got a right to rigorous assessment of the situation in coding theory** of the last decades. Apart from us, no one would ever dare to accept such a difficult decision to describe the real situation. **It is a pity!**

A discussion of all those measures that will restore the ethics of work in the science and will put things in order in the development of research on digital informatics at its new stage, we would leave it for authorized scientific structures.

The main OT publications can be found on our portals **www.mtdbest.ru** and **www.mtdbest.iki.rssi.ru** . Some of the more important of 600 blocks and articles we are listing below. You will find the other references there as well.

There are no sense to list thousands of books and articles on which to learn real methods of applied coding theory is completely impossible. Show vigilant yourself. Consult us. Keep developing the theory. **Please, learn to model!** There are no any new methods and technologies in the decoding algorithms science can be created without modeling.

And of course, we will continue to publish new reviews on the applied coding theory questions. Look for them aslo. It's very interesting and extremely useful. **It is a pity that there is no one to delegate this work to now at all.**

That's it! **You are ready for OT!** It is very simple. **Learn it.** Develop OT further. And let's to model all the algorithms !!! Only they will be the real results that should be presented to the contemporary scientific and technical community!

We wish you great successes!

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